# Torsional Oscillator. Part1.

**Physics 401. Fall 2016**
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# Transients in a Torsional Oscillator

- **Electrical RLC circuits**
- **Torsional Oscillator**
- **Damping**
- **Data Analysis**

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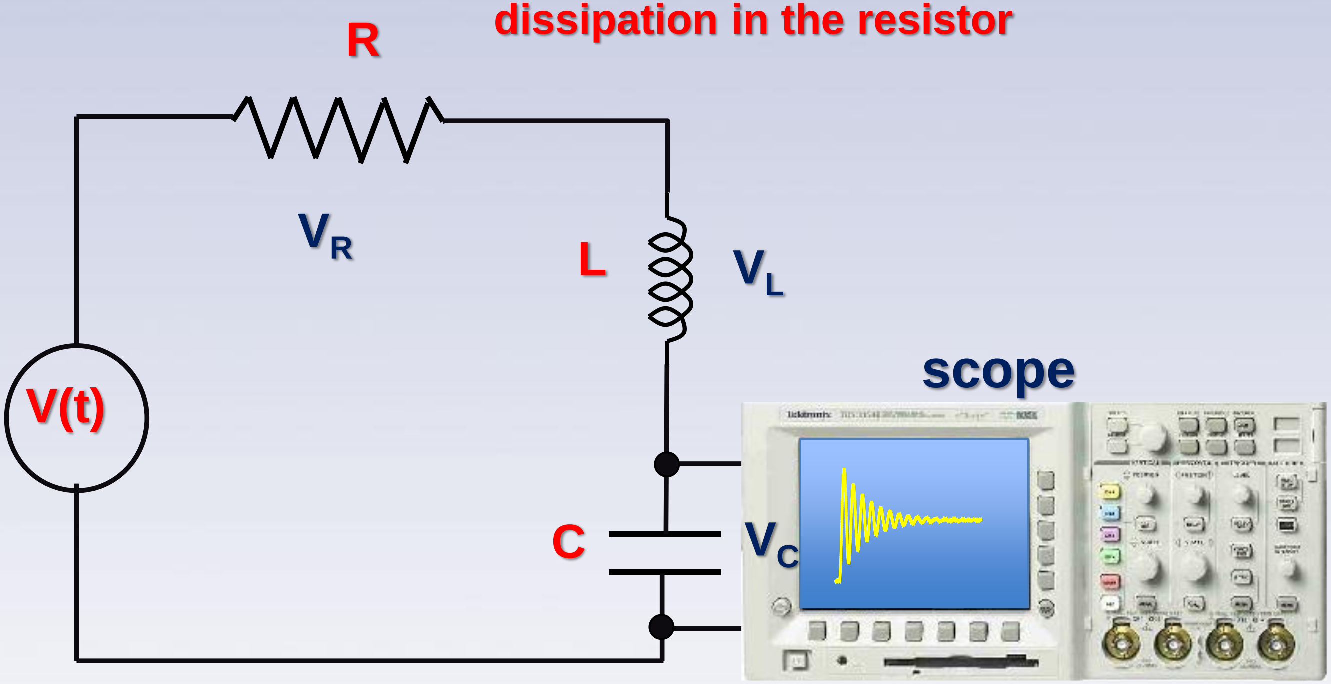
# Transients in RLC circuit.

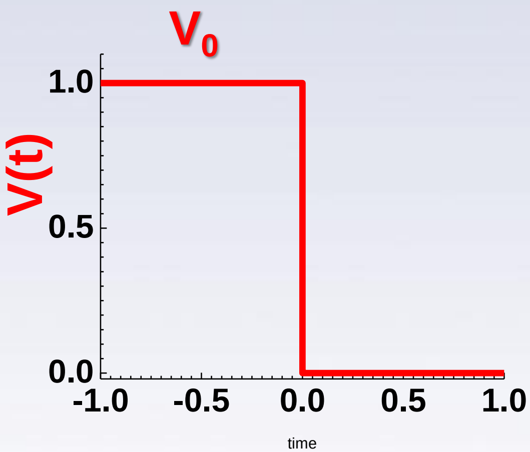
$V_R+V_L+V_C=V(t)$

If $V(t)=0$

$$L\frac{d^2}{dt^2}q(t)+R\frac{d}{dt}q(t)+\frac{q(t)}{C}=0\ ,\quad \frac{q(t)}{C}=V_0$$

Damping term. Reflects energy dissipation in the resistor

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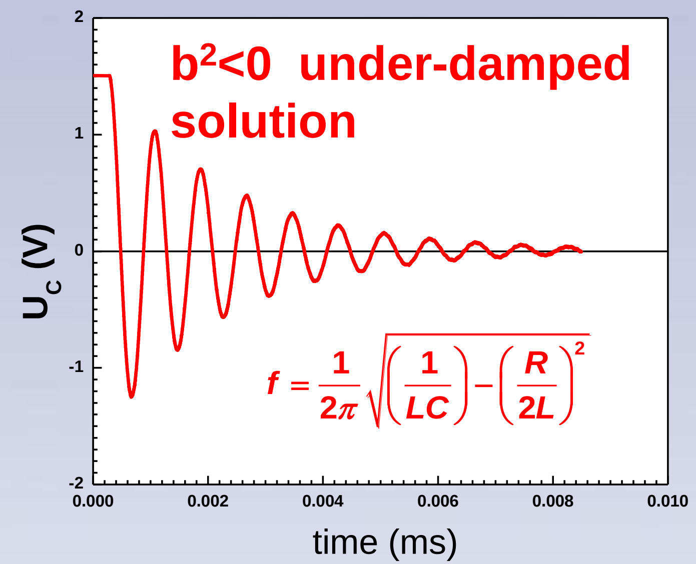
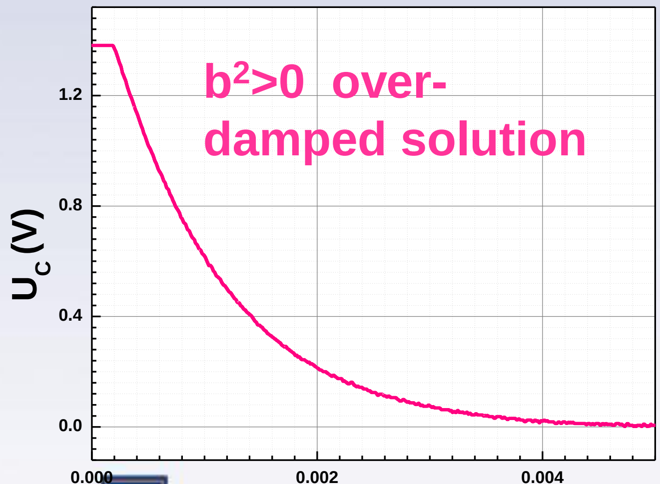
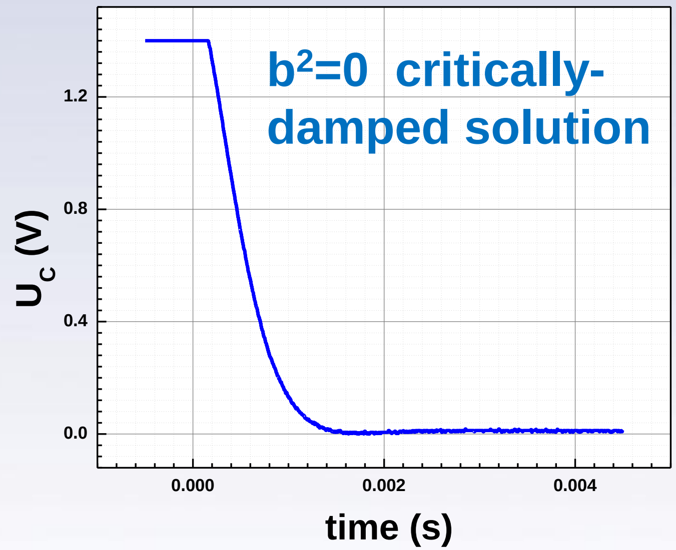
# RLC: three solutions.

$$a = \frac{R}{2L} \quad , \quad b = \sqrt{\left(\frac{R}{2L}\right)^2 - \left(\frac{1}{LC}\right)}$$

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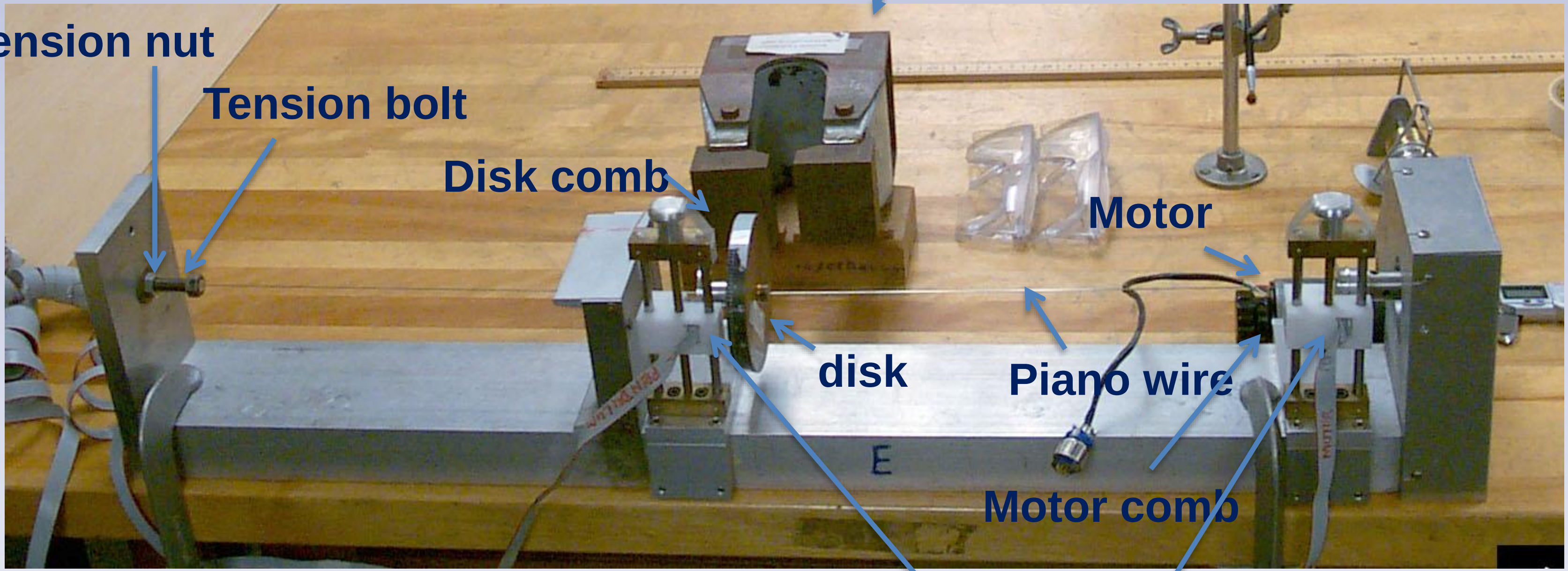
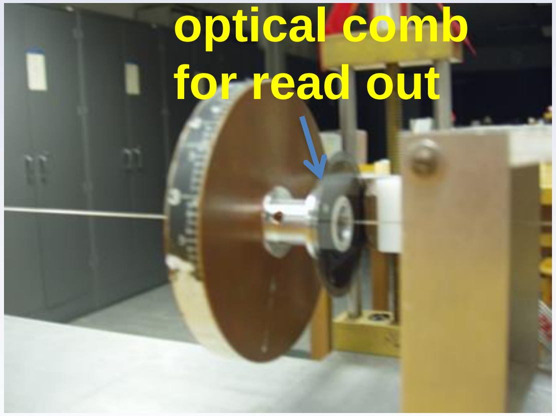
# The Torsional Oscillator.

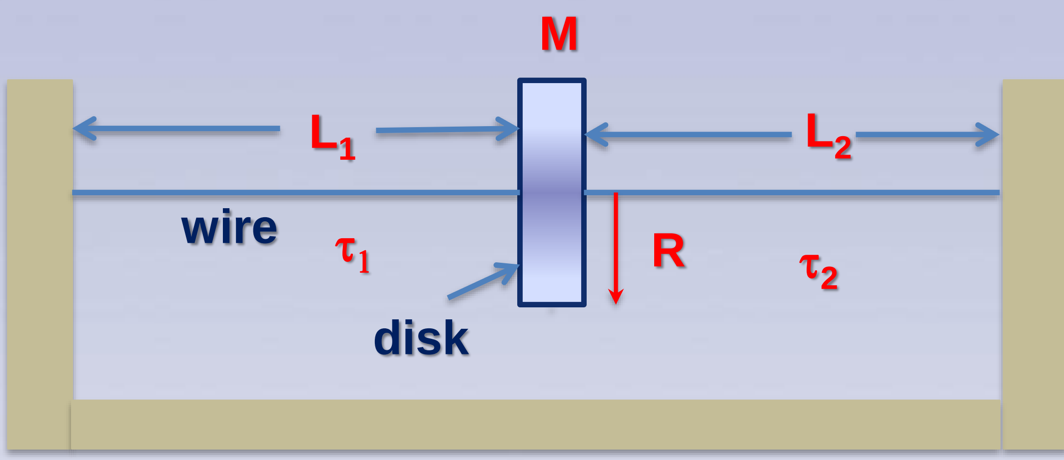



**Momentum of Inertia I for disk with radius R and mass M:**

$$I = \frac{MR^2}{2}$$

# The Torsional Oscillator.




**Wires 1 and 2 exert the torques $\tau_1$ and $\tau_2$ on the disk of mass M**

$$\tau = \tau_1 + \tau_2 = -K_1\theta - K_2\theta = -K\theta$$

$$K_1 = \frac{\pi G r^4}{2L_1}$$

*$\theta$ : angular deflection of the disk*
*r : radius of the wires*
*$L_i$: length of the wire I*
*G: shear modulus of the wire*

*A typical shear modulus for steel is $8.3 \times 10^{10}$ N/m$^2$*

$$K = K_1 + K_2 = G\frac{\pi}{2}r^4\left(\frac{1}{L_1} + \frac{1}{L_2}\right)$$

*K – torsional spring constant*

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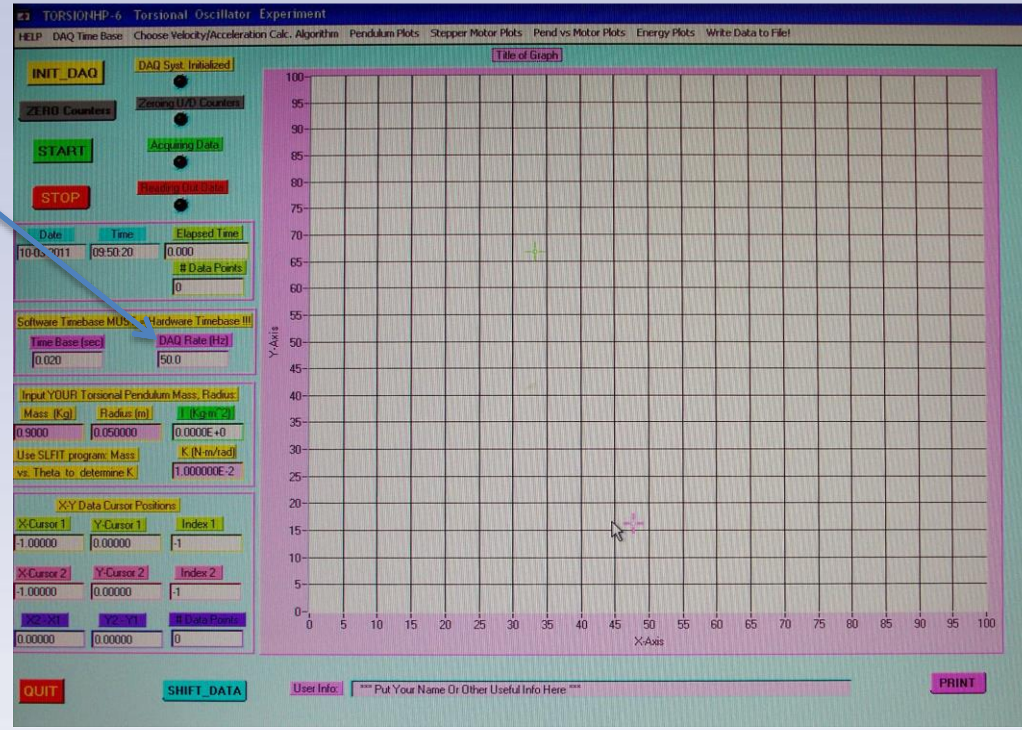
# The Data Acquisition Setup and Program

DAQ rate (Hz)




Interface card

Program window

**Program can accept only 10000 points. If sampling rate is 50Hz – the maximum time of data collection is 200s!**

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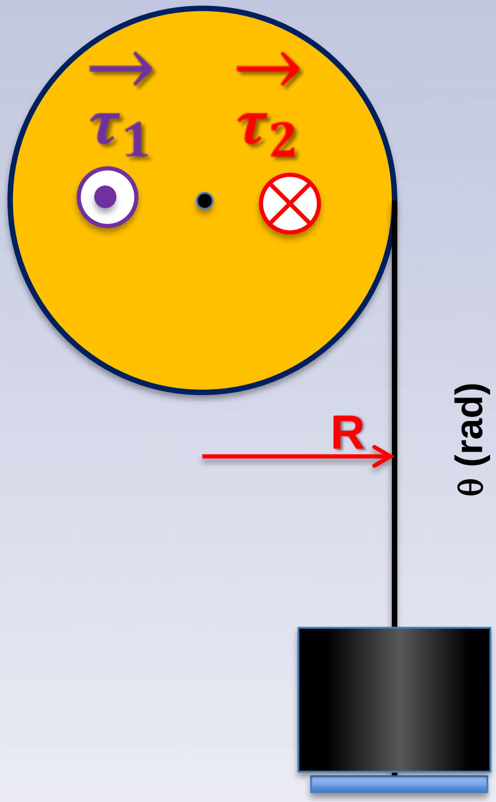
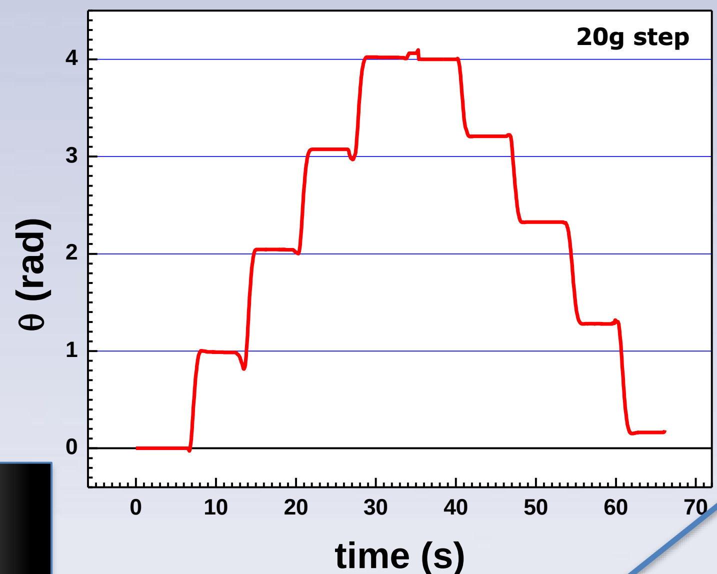
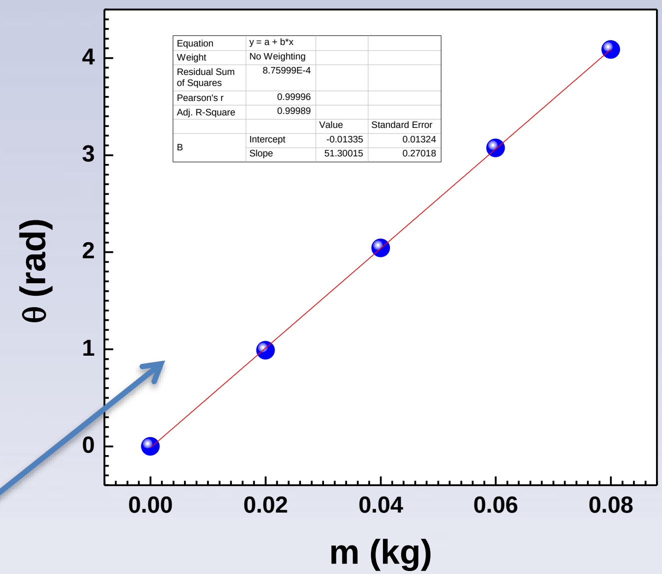
# Measuring of the Torsional Spring constant.

$$\vec{\tau}_1 + \vec{\tau}_2 = 0 \longrightarrow K\theta = mgR$$




| | | | |
|---|---|---|---|
| Equation | y = a + b*x | | |
| Weight | No Weighting | | |
| Residual Sum of Squares | 8.75999E-4 | | |
| Pearson's r | 0.99996 | | |
| Adj. R-Square | 0.99989 | | |
| | | Value | Standard Error |
| B | Intercept | -0.01335 | 0.01324 |
| | Slope | 51.30015 | 0.27018 |

slope

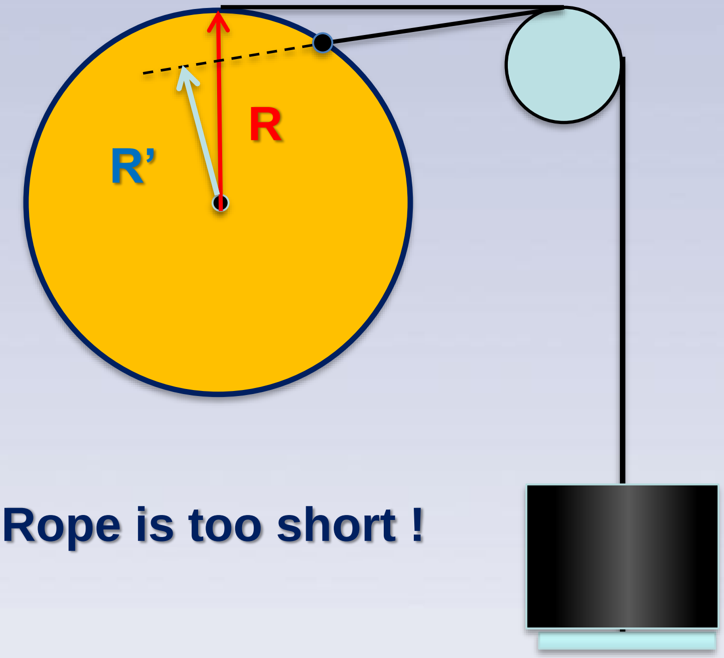
$$\theta = \frac{gR}{K} m \qquad K = \frac{gR}{slope}$$

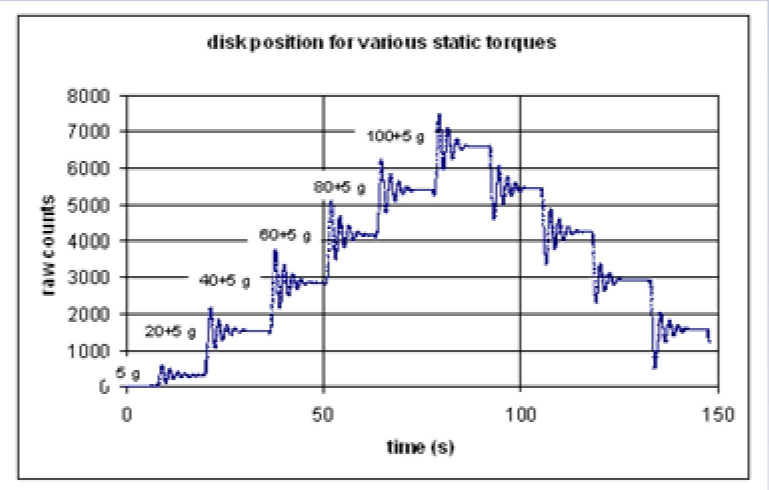
$g = 9.81 m/s^2$

*Slope=51.3rad/kg*

K=0.00971Nm/rad

# Measuring of the Torsional Spring constant. Possible problems.




**Rope is too short !**

$$\tau = R \times F$$

**Avoid the over damping of the pendulum motion and any extra sources of friction.**

# Torsional Pendulum. Scientific application.

## Measuring of the electrostatic forces.

Charles-Augustin de Coulomb
1736-1806

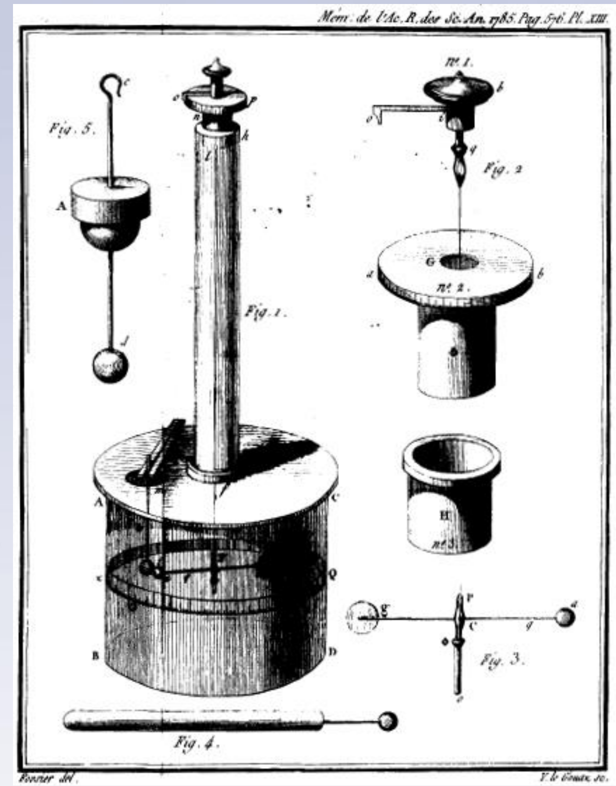
$$\vec{\tau}_1 + \vec{\tau}_2 = 0$$

$$K\theta = FL;$$

Where F is electrostatic force and L is the length of the balance beam.

## Coulomb's law

$$F = k_e \frac{q1 * q2}{r^2}; \quad k_e = \frac{1}{4\pi\varepsilon_0}$$




**Coulomb's torsion balance.**
Courtesy of Wikipedia

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# Torsional Pendulum. Scientific application.

Henry Cavendish
(1731–1810)

## Measuring of the gravitational forces.

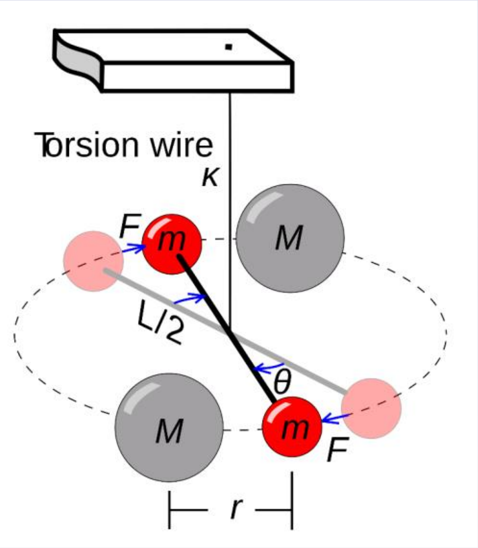
$$\vec{\tau}_1 + \vec{\tau}_2 = 0$$

$$K\theta = FL;$$

Where F is gravitational force and L is the length of the balance beam.

Gravitational Law

$$F = \frac{GmM}{r^2}$$




Cavendish torsion balance experiment.
Courtesy of Wikipedia

Cavendish's result

$G = 6.74 \times 10^{-11}$ m$^3$ kg$^{-1}$ s$^{-2}$,

Currently accepted value

$6.67428 \times 10^{-11}$ m$^3$ kg$^{-1}$ s$^{-2}$.

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# The Torsional Oscillator. "No damping".

$$\tau = \tau_1 + \tau_2 = -K_1\theta - K_2\theta = -K\theta$$

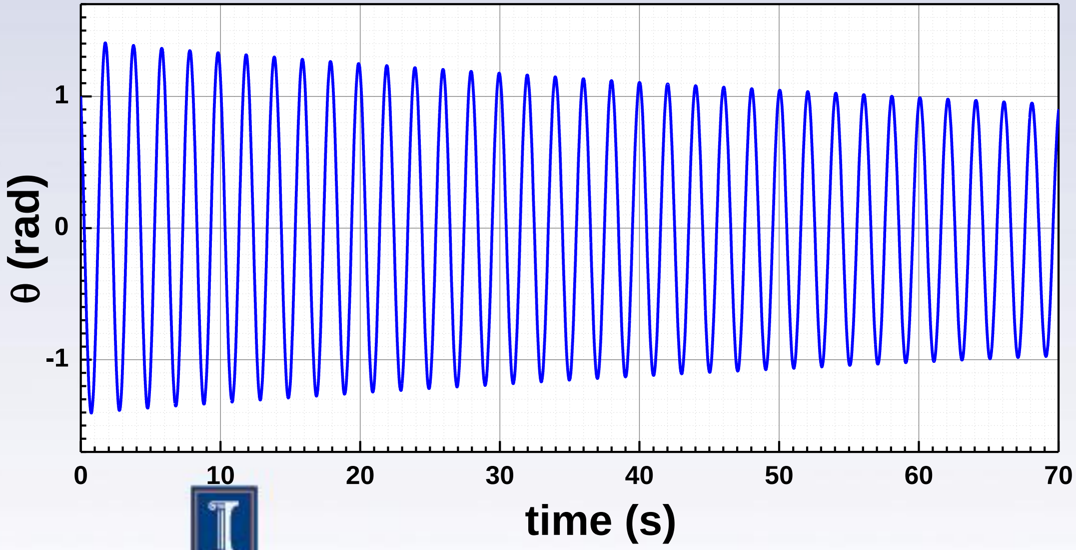
$$K_1 = \frac{\pi G r^4}{2L_1}; \quad K = K_1 + K_2 = \frac{\pi G r^4}{2}\left(\frac{1}{L_1} + \frac{1}{L_2}\right)$$

**If there is no dissipation:**

$$I\frac{d^2\theta}{dt^2} = -K\theta$$

**Solution:** $\theta = \theta_0 \sin(\omega_0 t + \phi)$ **with** $\omega_0 = \sqrt{\frac{K}{I}}$

**If we know I we can calculate K**




**From time trace $\theta(t)$ we can find $\omega_0$ it can be done by measuring period but better (and faster!) to perform the nonlinear fitting.**

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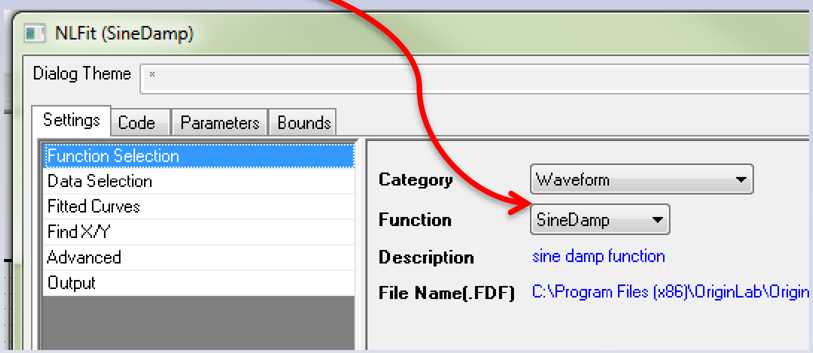
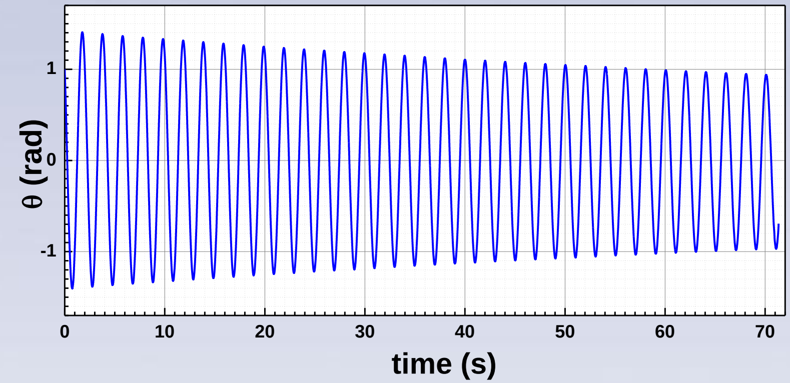
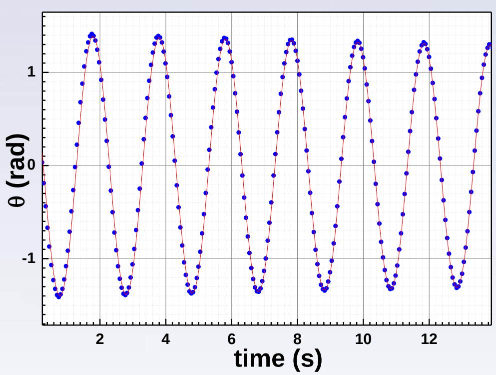
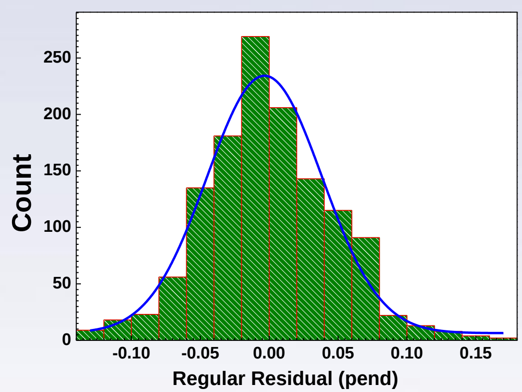
# The Torsional Oscillator. "No damping". Fitting

**"No damping" is not realistic situation fitting should be done to SineDamp function** $y = y_0 + A\exp\left(\frac{-x}{t_0}\right)\sin\left(\pi\frac{(x-x_c)}{w}\right)$ $\quad \omega_0 = \frac{\pi}{w}$




**SineDamp:** $y = y_0 + A\exp\left(\frac{-x}{t_0}\right)\sin\left(\pi\frac{(x-x_c)}{w}\right)$

| | Value | Standard Error |
|---|---|---|
| y0 | -0.0024 | 0.0013 |
| xc | -0.7236 | 9.3E-4 |
| w | 1.00517 | 2.5E-5 |
| t0 | 178.02 | 2.44 |
| A | 1.409 | 0.004 |

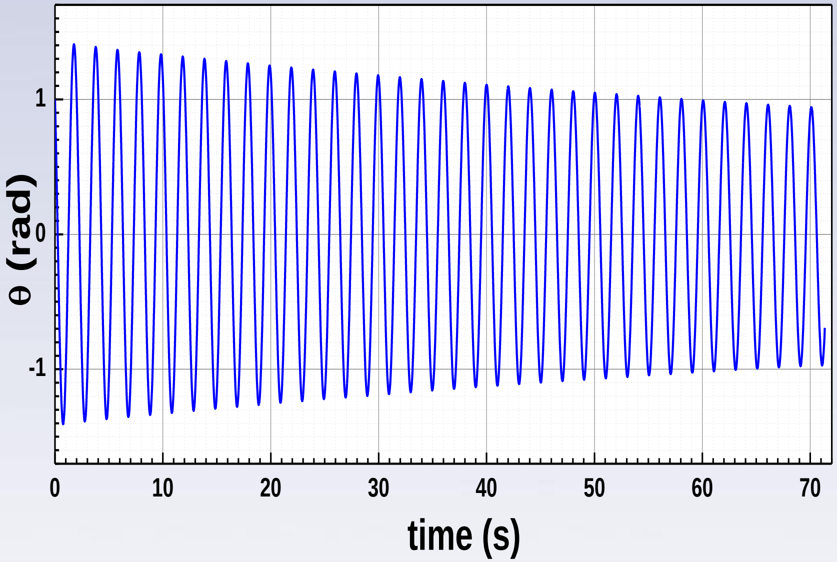
$$K = \omega_0^2 I \approx 1.12 \times 10^{-2}\frac{Nm}{rad}$$

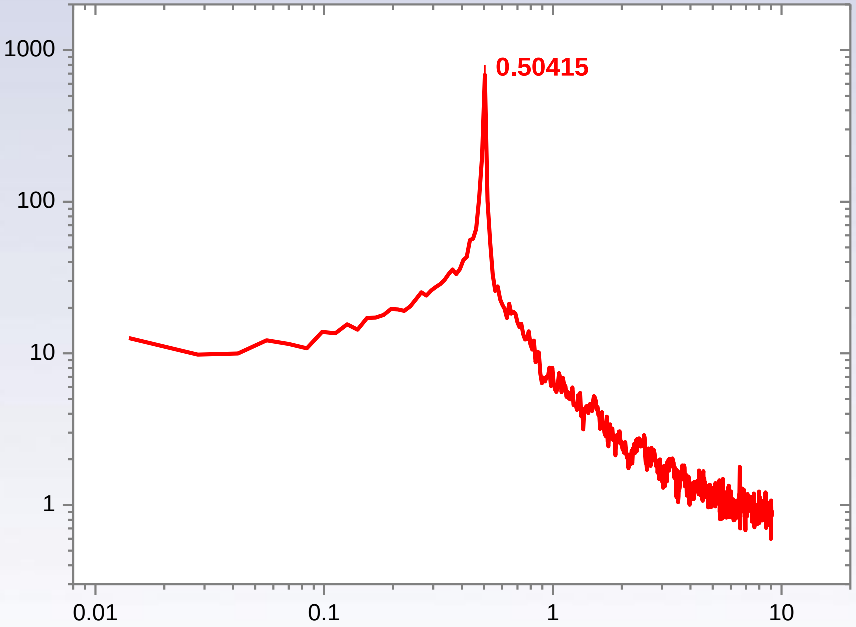
# The Torsional Oscillator. "No damping". Fitting.

$$\omega_0 = \frac{\pi}{w} \qquad f_0 = \frac{1}{2w}$$

***From "SineDamp fitting $f_0$=0.497Hz***

**Resonance frequency can be also found by applying FFT on the raw data**

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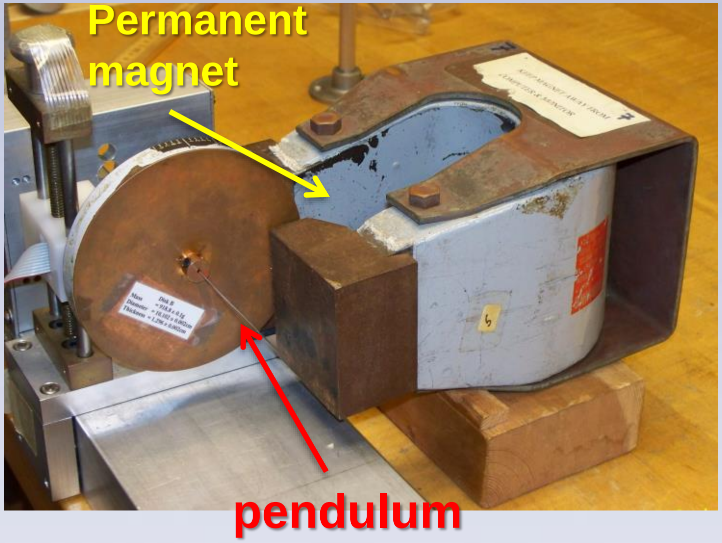
# Viscous (magnetic) damping.

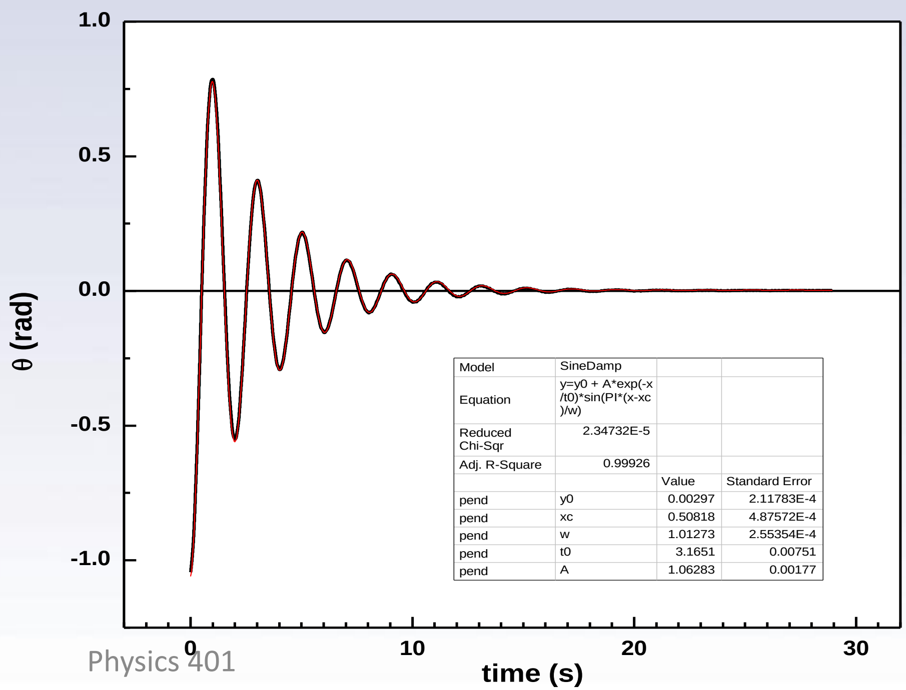



$$I\frac{d^2\theta}{dt^2} + K\theta + R\frac{d\theta}{dt} = 0$$

Damping term

**The solutions are exactly the same as in case of RLC circuit (three solutions)**

**Under damped case**

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# Viscous damping. Logarithmic decrement.

$$I\frac{d^2\theta}{dt^2} + K\theta + R\frac{d\theta}{dt} = 0$$

$$\delta = ln\left(\frac{\theta_{n+1}}{\theta_n}\right);$$

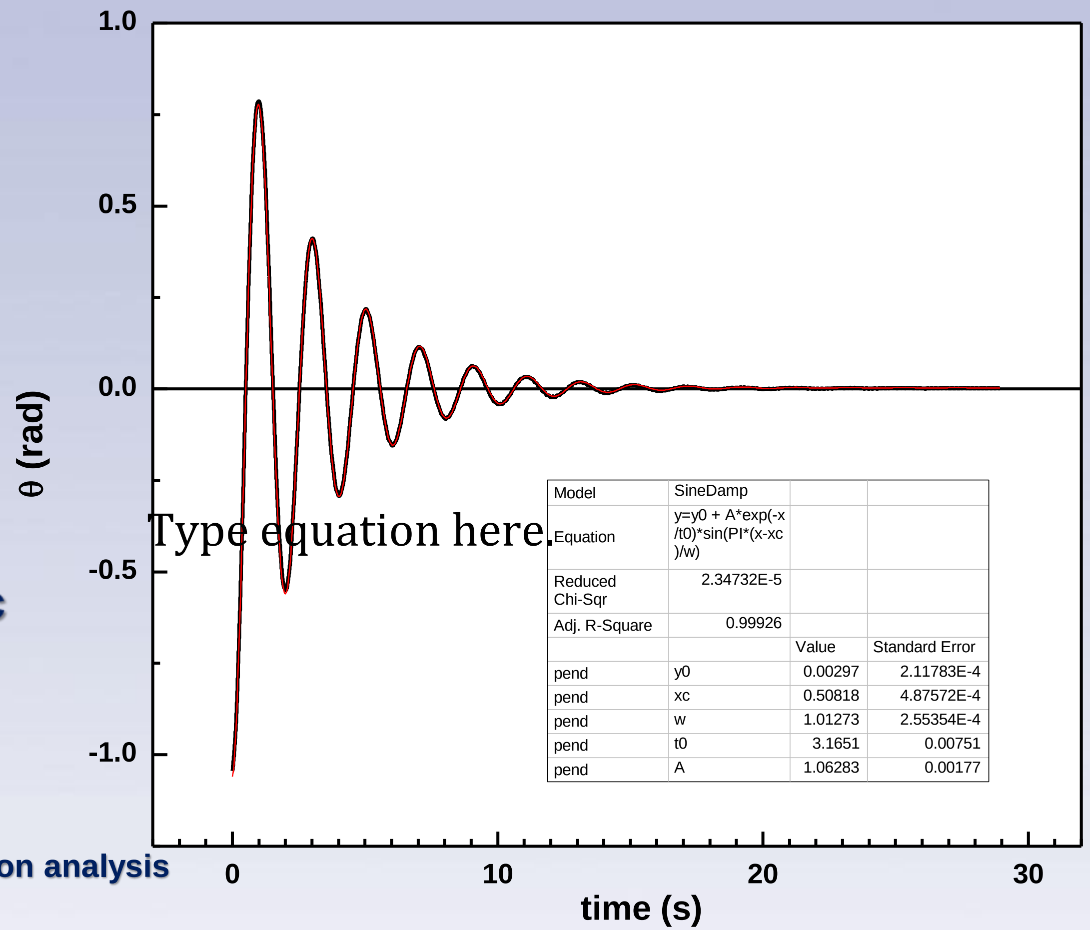
**For viscous damping** $\delta = \frac{T}{t_0}$**;**

**where**

**T – period and $t_0$ – characteristic decay time***

**from SineDamp fitting function**

**T=2w and** $\delta = \frac{2w}{t_0}$




**from error propagation analysis**

**δ=0.640±0.002**

**From *SineDamp* fitting exponential decay term is** $\exp\left(\frac{-t}{t_0}\right)$

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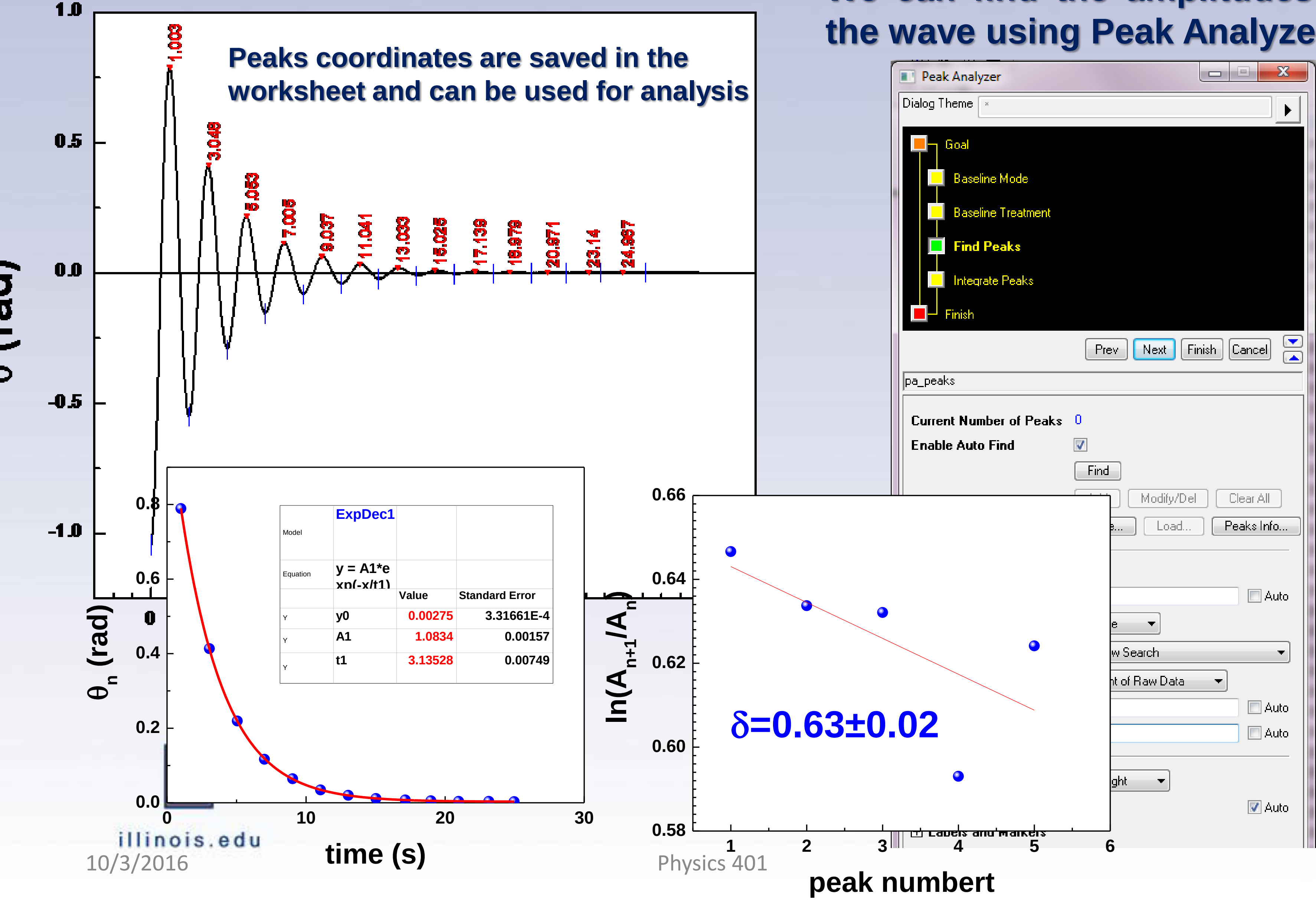
**$*a=\frac{1}{t_0}$ (write up)**

# Viscous damping. Logarithmic decrement.

**We can find the amplitudes of the wave using Peak Analyzer**

**Peaks coordinates are saved in the worksheet and can be used for analysis**




| Model | ExpDec1 | | |
|---|---|---|---|
| Equation | y = A1*exp(-x/t1) | | |
| | | Value | Standard Error |
| Y | y0 | 0.00275 | 3.31661E-4 |
| Y | A1 | 1.0834 | 0.00157 |
| Y | t1 | 3.13528 | 0.00749 |

# Analysis. 1

1. Fitting to damp exponential decay function. Outcome: resonance frequency and decrement coefficient .
2. Applying FFT procedure. Result – resonance frequency.
3. Using Origin Peak Analyzer we can find amplitudes and positions of the damped sine wave maximum end then plot the envelope.
4. You can directly obtain the envelope of the damped sine wave by using Origin (optional).

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# Coulomb damping. Theory

$$I\ddot{\theta} + K\theta + \tau_{Coulomb} = 0$$

$$\tau_{Coulomb} = C\frac{|\dot{\theta}|}{\dot{\theta}}$$

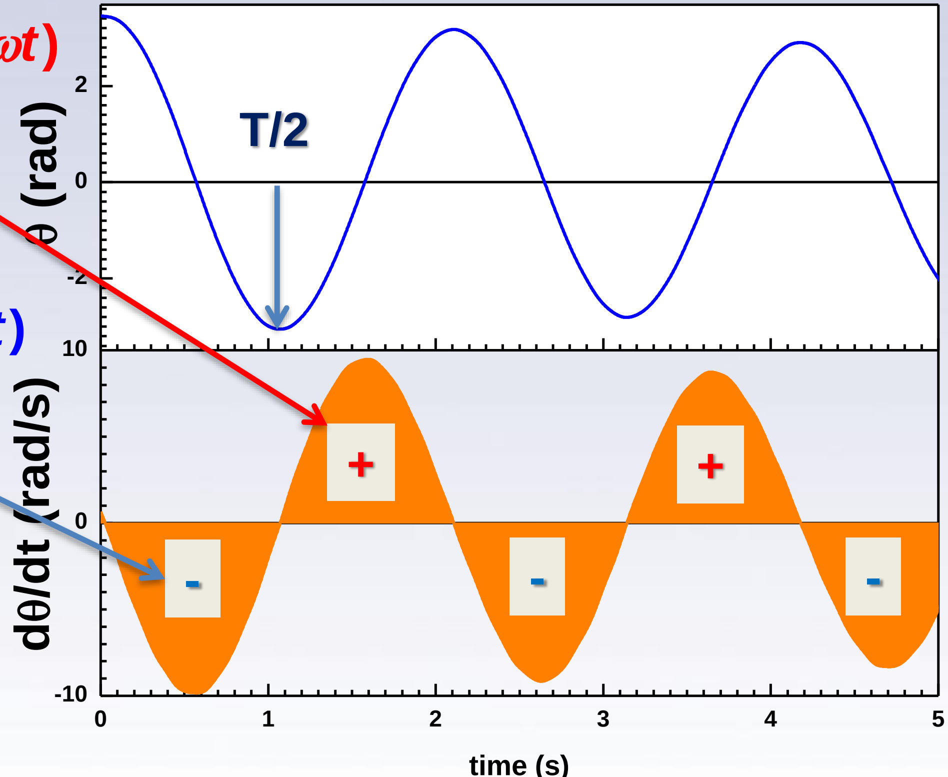
Amplitude decreases by 4C/K per period linearly !

$$\theta(t) = +C/K + (\theta_0 - (4n-1)C/K)\cos(\omega t)$$

$$(n-\frac{1}{2})T \le t \le nT \quad n = 1,2,...$$

$$\theta(t) = +C/K + (\theta_0 - (4n-3)C/K)\cos(\omega t)$$

$$(n-1)T \le t \le (n-\frac{1}{2})T \quad n = 1,2,...$$

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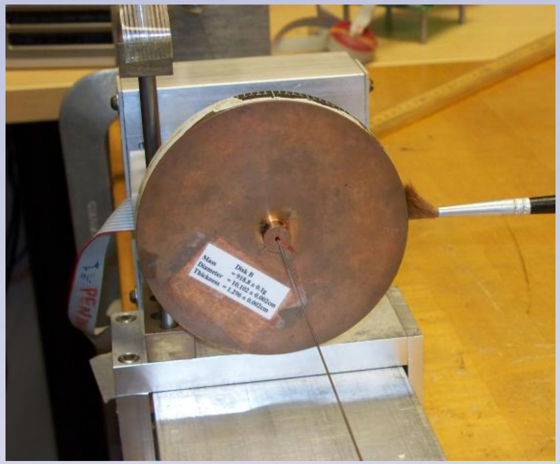
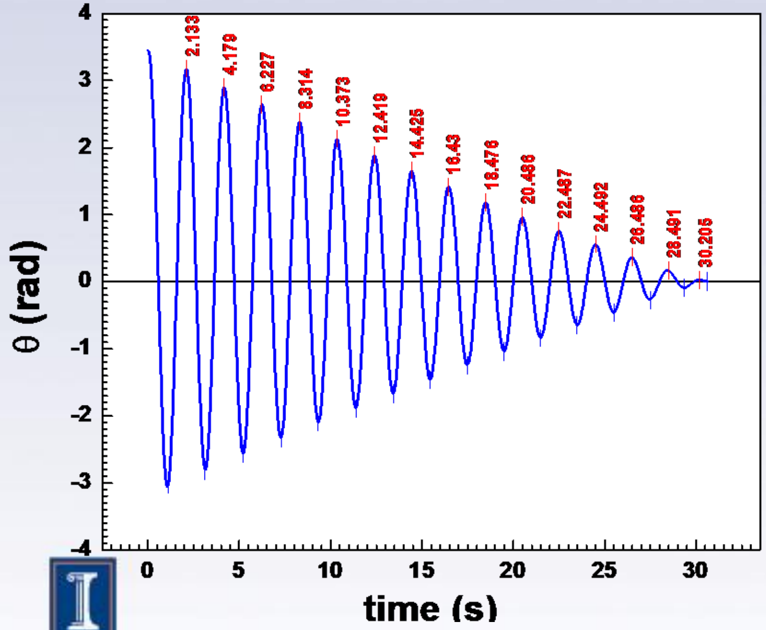
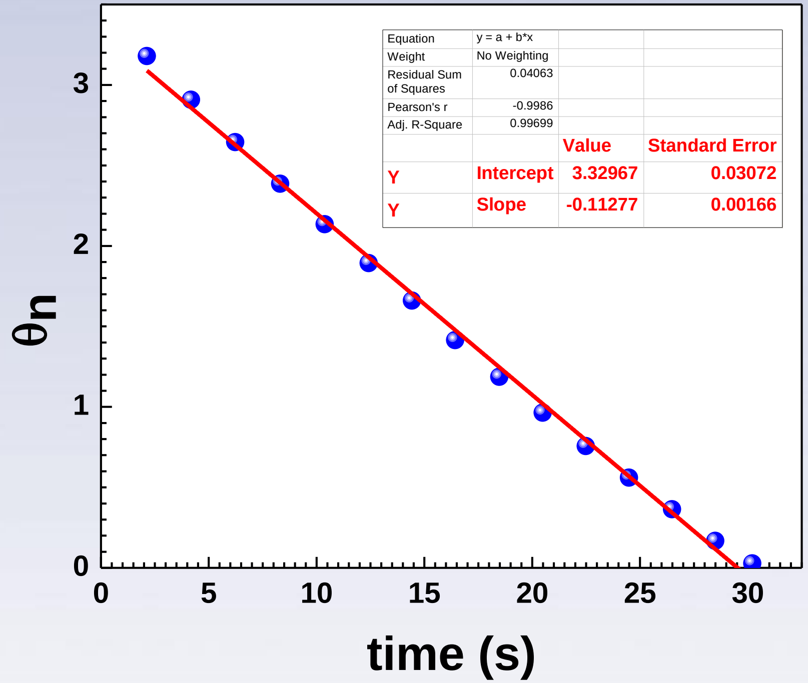
# Coulomb damping. Experiment



**Amplitude decreases by 4C/K per period linearly !**




| | | | |
|---|---|---|---|
| Equation | y = a + b*x | | |
| Weight | No Weighting | | |
| Residual Sum of Squares | 0.04063 | | |
| Pearson's r | -0.9986 | | |
| Adj. R-Square | 0.99699 | | |
| | | Value | Standard Error |
| Y | Intercept | 3.32967 | 0.03072 |
| Y | Slope | -0.11277 | 0.00166 |

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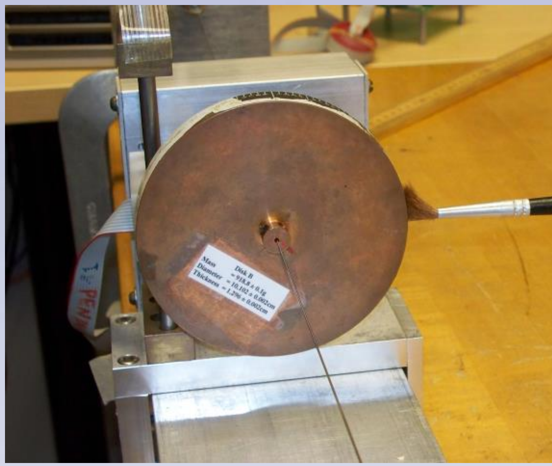
# Coulomb damping. Experiment

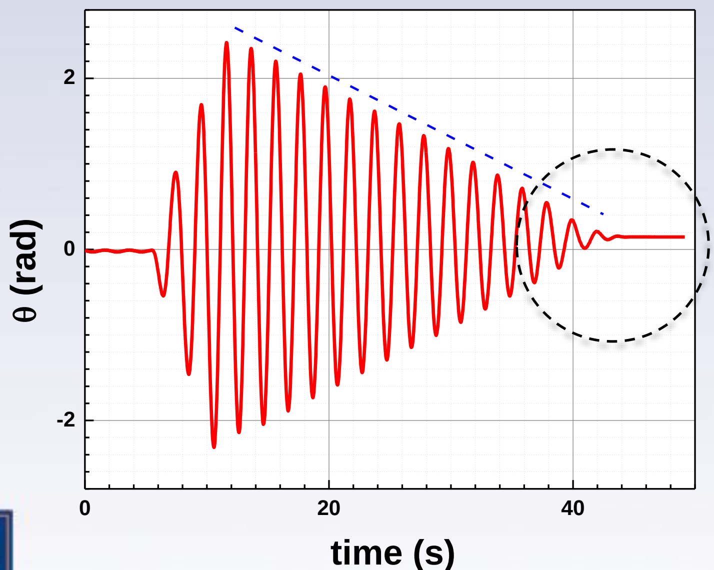


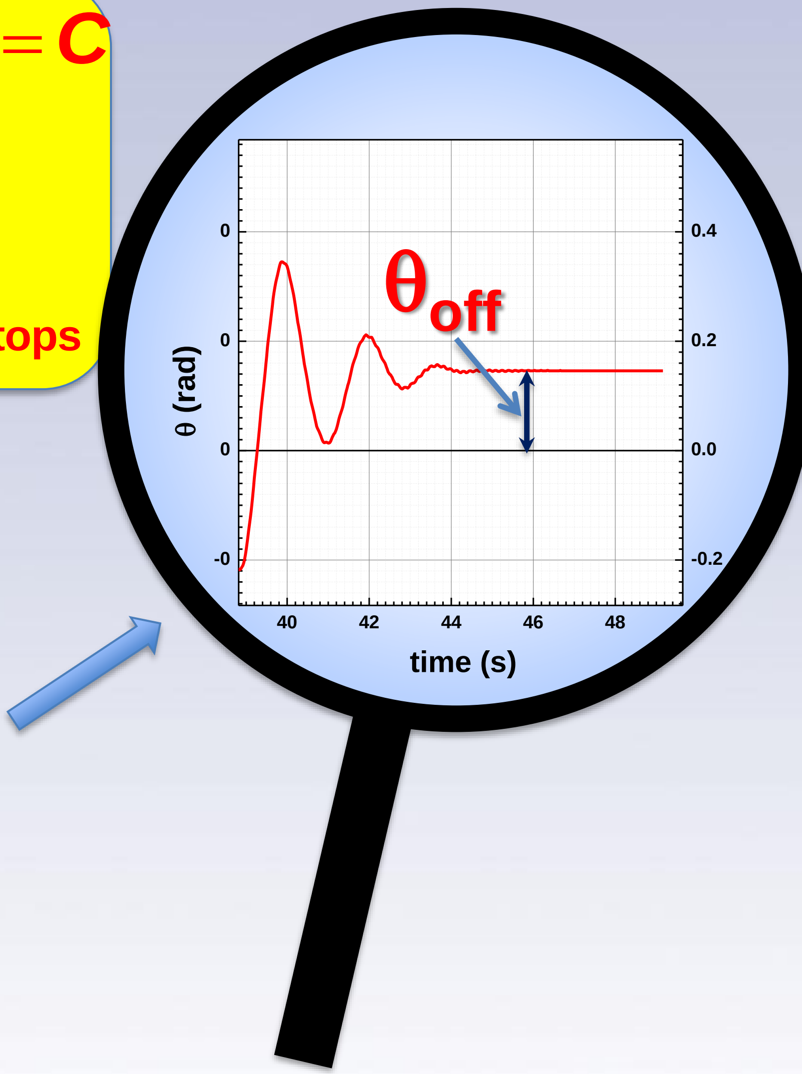
$$|\tau_{Coulomb}| = C$$

$$K\theta \sim \theta$$

*if* $K\theta \leq C$

**pendulum stops**

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# Turbulent damping. Theory

$$I\ddot{\theta}+K\theta+\tau_{Turb}=0$$

$$\tau_{turb}=C_t\,\mathrm{sgn}(\dot{\theta})\left|\dot{\theta}\right|^n$$

In case of n=1 → viscous damping

Logarithmic decrement in case of turbulent damping is no more constant and in case n=2 can be calculated as $\delta=\frac{8C}{3I}\theta_0$

Expected result – decrement decreases with decreasing of the amplitude

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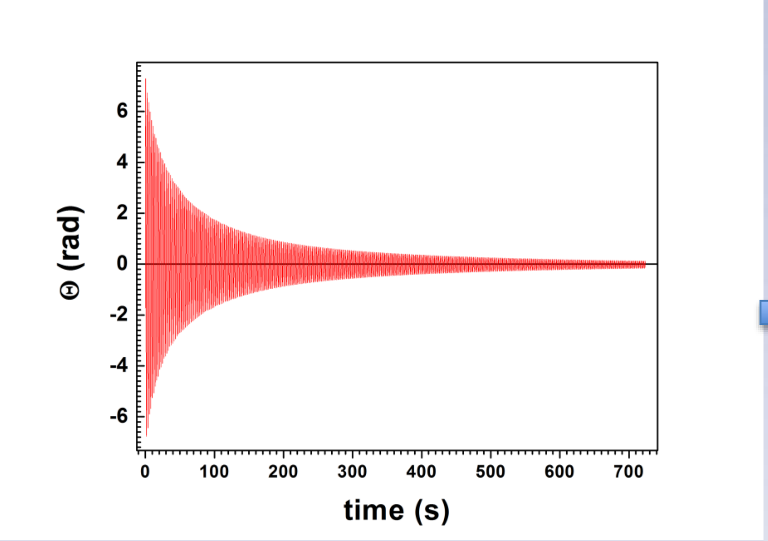
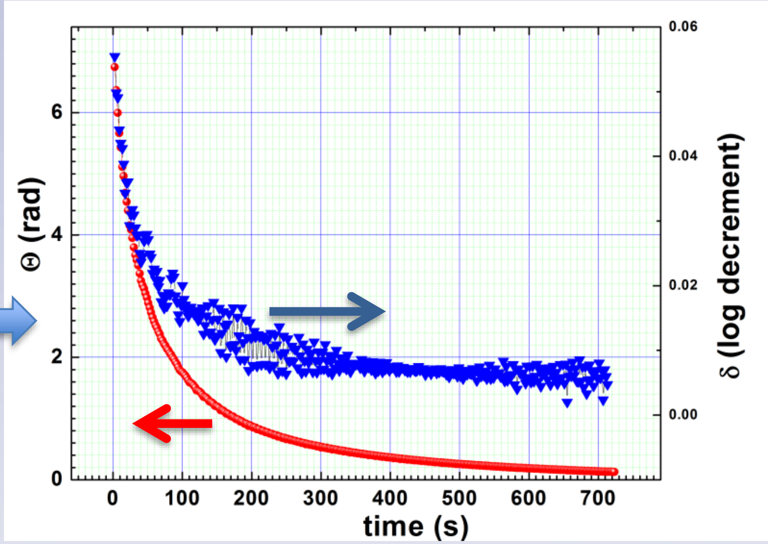
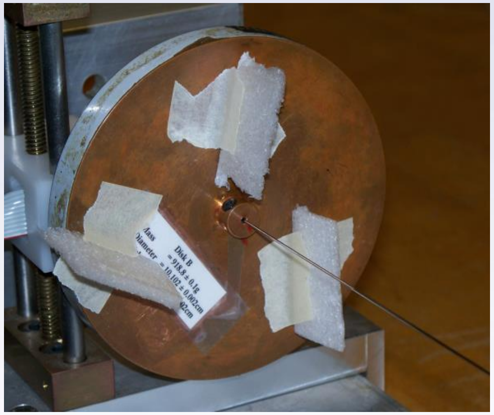
# Turbulent damping. Experiment

**Analyzing the envelope of the damped oscillating time record we can calculate the log decrement factor**



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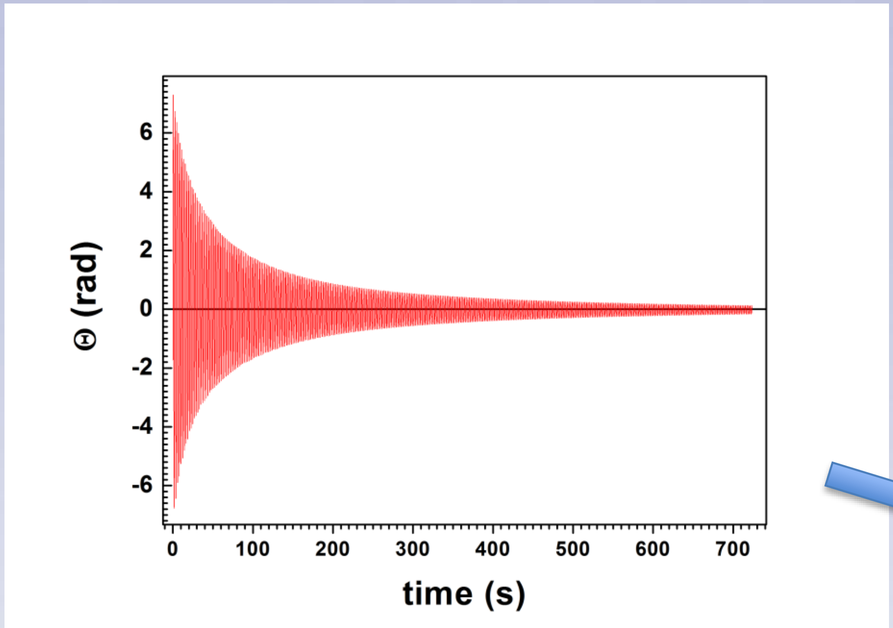
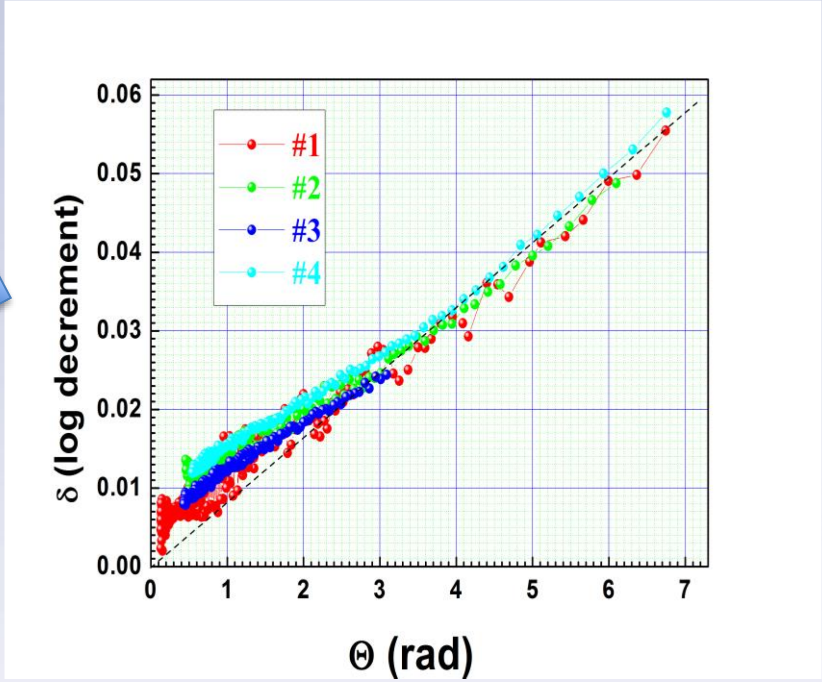
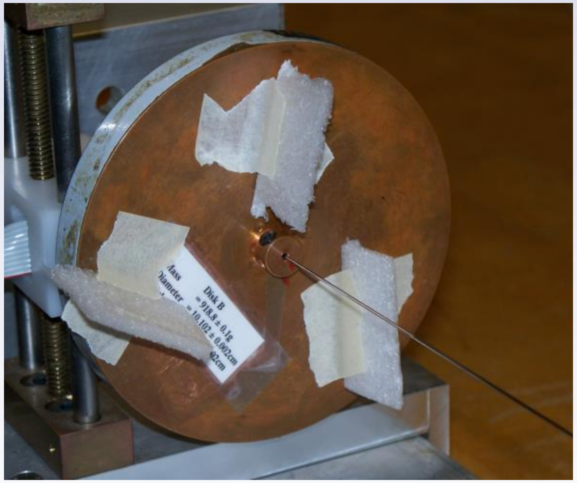
# Turbulent damping. Experiment.



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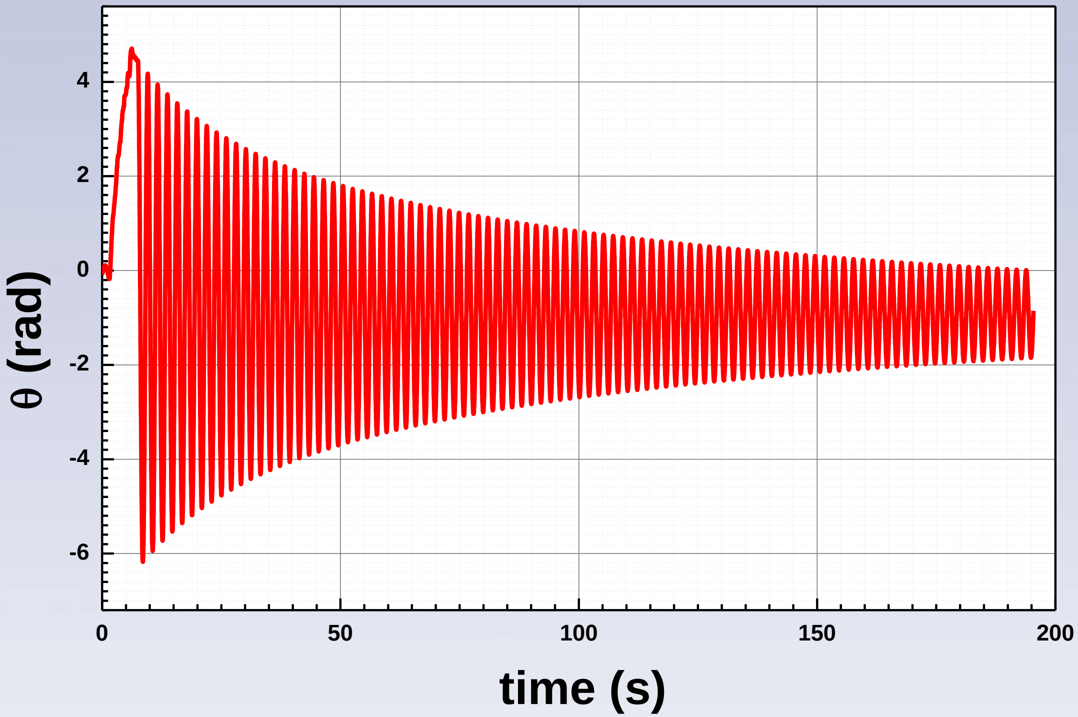
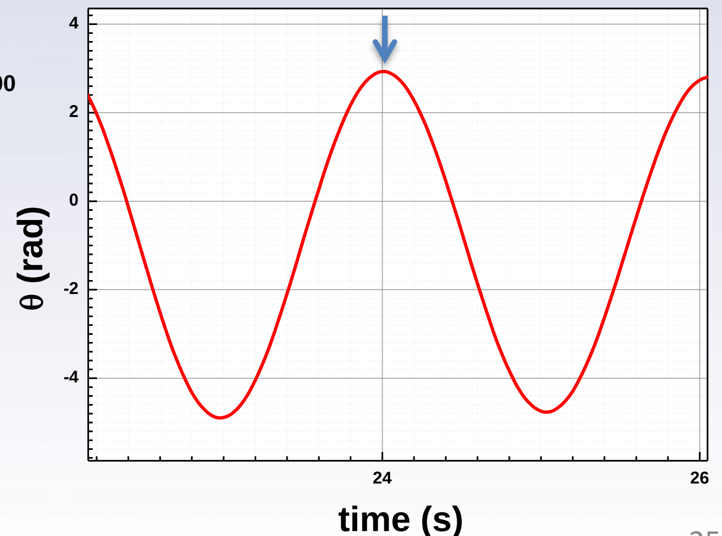
# Data analysis. Finding the peaks.




**Raw data.**

**Our goal:** find the positions and amplitudes of the peaks

$X_i, Y_i$




**1st Technique: using "*FindPeaks*" option**

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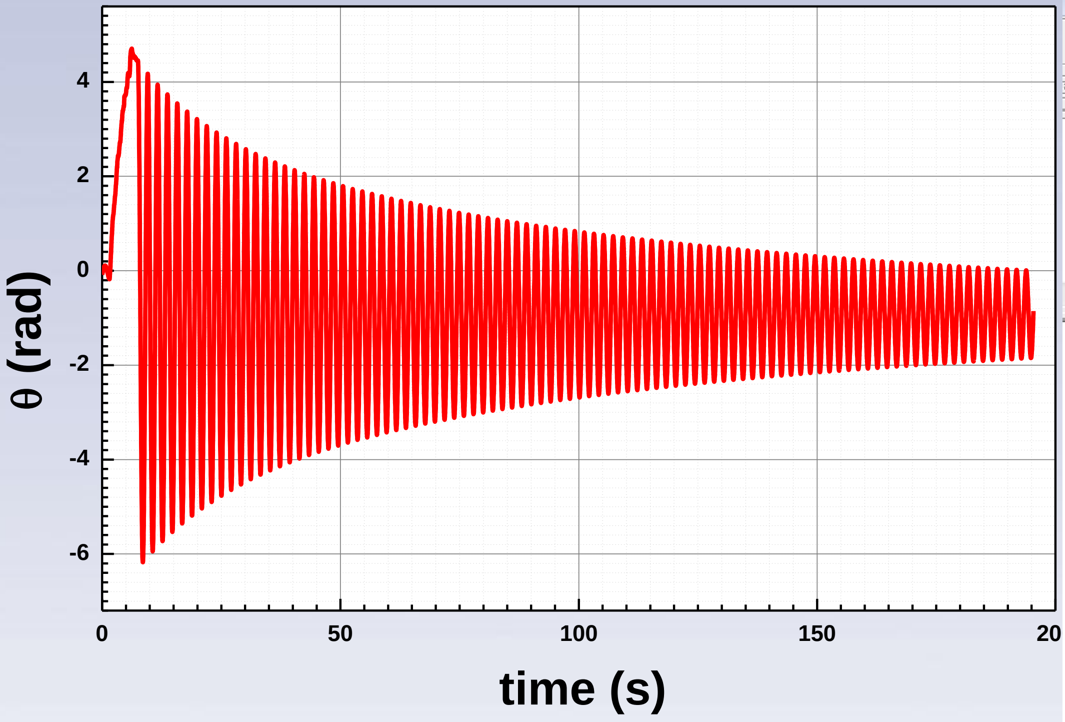
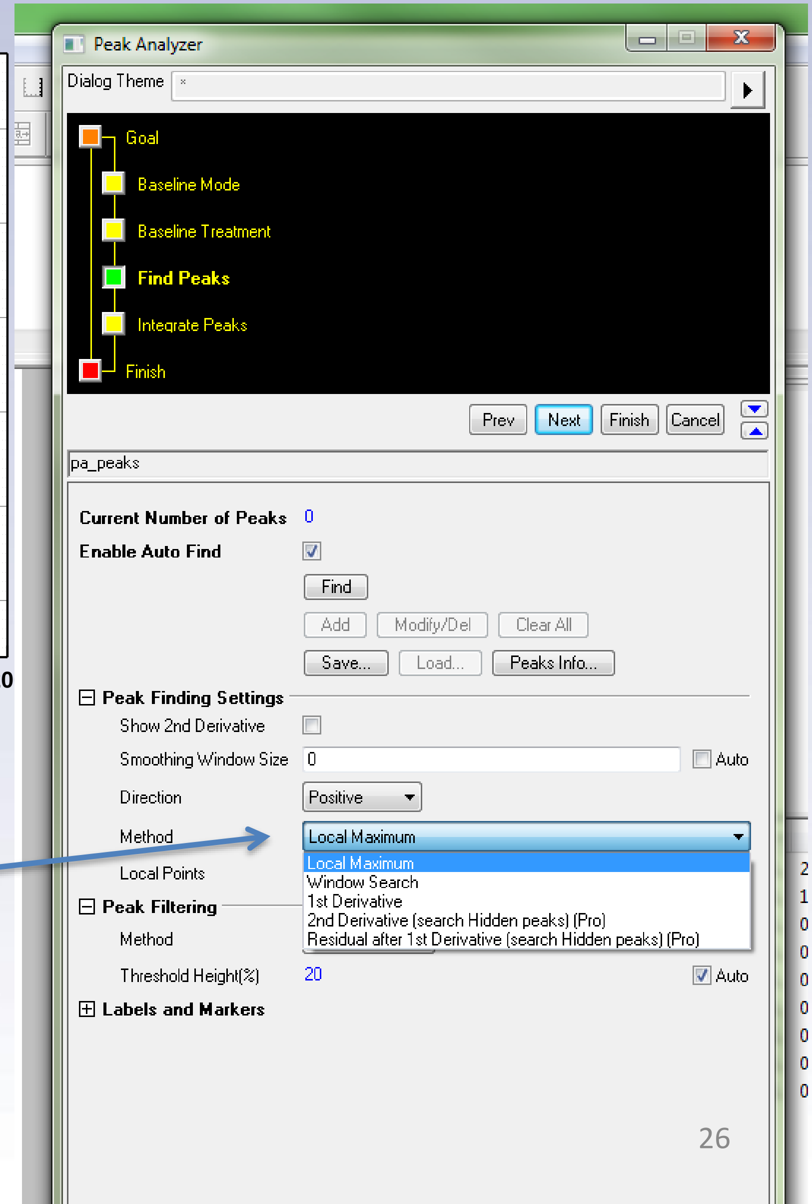
# Data analysis. Finding the peaks.




***Local Maximum*** **works well for not noisy oscillating dependencies**

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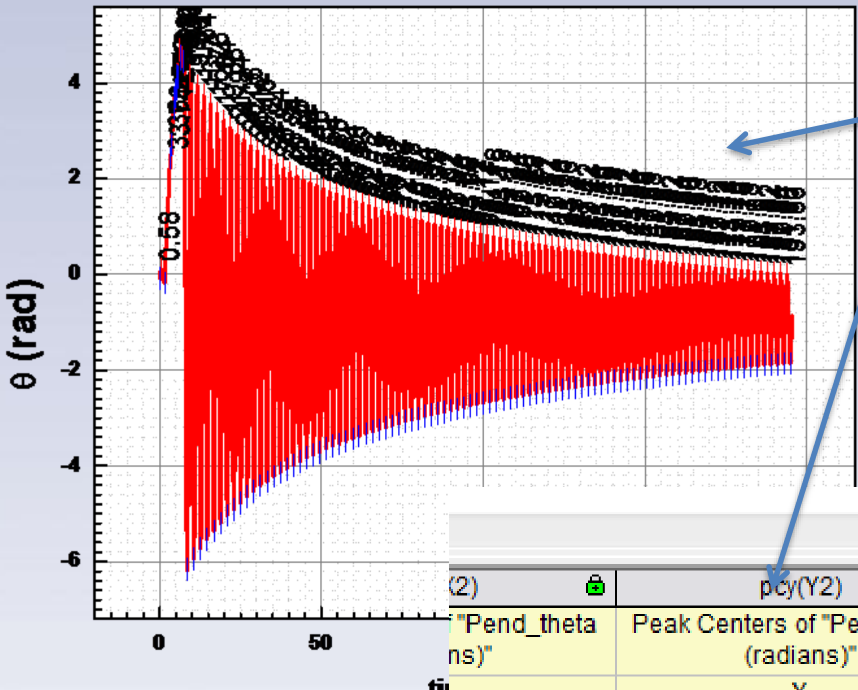
# Data analysis. Finding the peaks.




New plot + labels as a result of finding the peaks

"Peaks" data can be found in a Worksheet and using this data you can plot the dependence of amplitude on time

| (X2) "Pend_theta ns)" | pcy(Y2) Peak Centers of "Pend_theta (radians)" Y | pmx(X3) Base Markers of "Pend_theta (radians)" X | pmy(Y3) Base Markers of "Pend_theta (radians)" Y |
|---|---|---|---|
| 0.58 | 0.10738 | 0 | -0.0813 |
| 3.36 | 2.44056 | 1.54 | -0.17948 |
| 3.76 | 2.72742 | 1.54 | -0.17948 |
| 4.54 | 3.4829 | 3.44 | 2.42676 |
| [illegible] | [illegible] | 3.44 | 2.42676 |
| [illegible] | [illegible] | [illegible] | [illegible] |
| 4.88 | 3.72834 | [illegible] | [illegible] |
| 5.1 | 3.87177 | 4.54 | 3.4829 |
| 5.16 | 3.87253 | 4.54 | 3.4829 |

"Positive" peaks

"Negative" peaks

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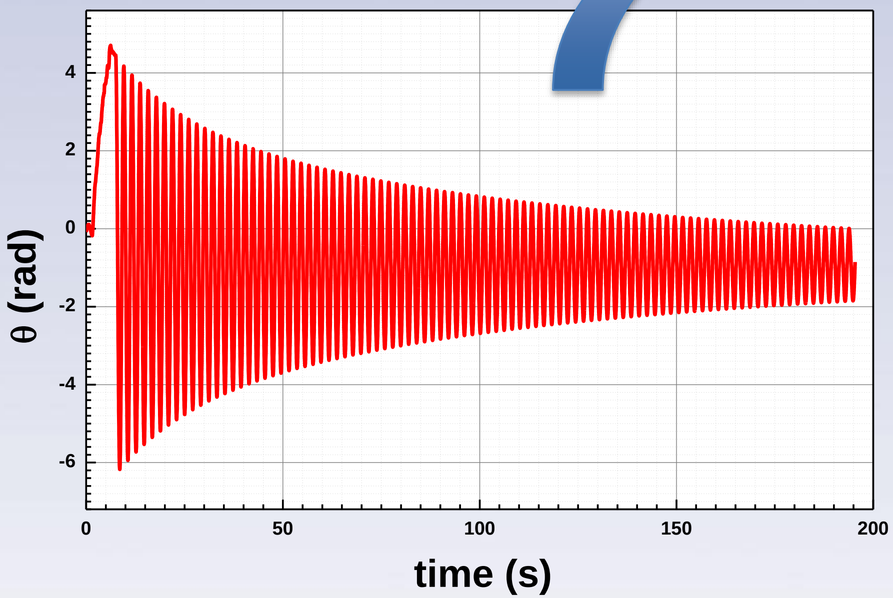
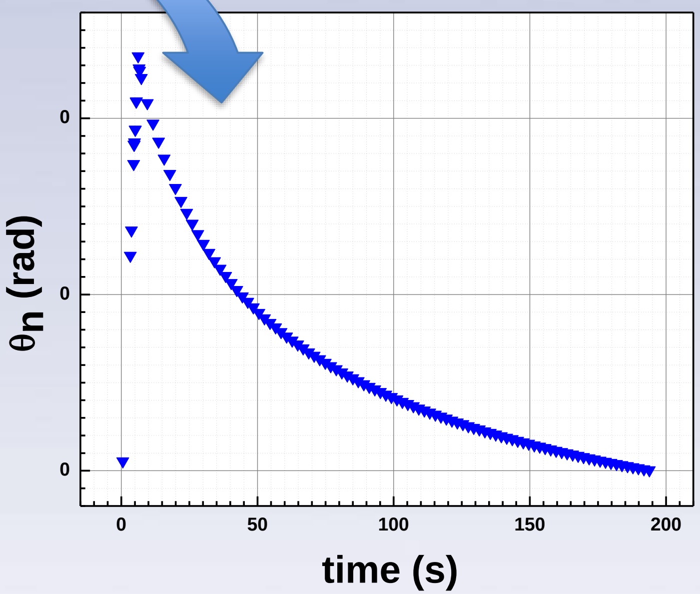
# Data analysis. Finding the peaks.

**Original Data**




**Envelope**

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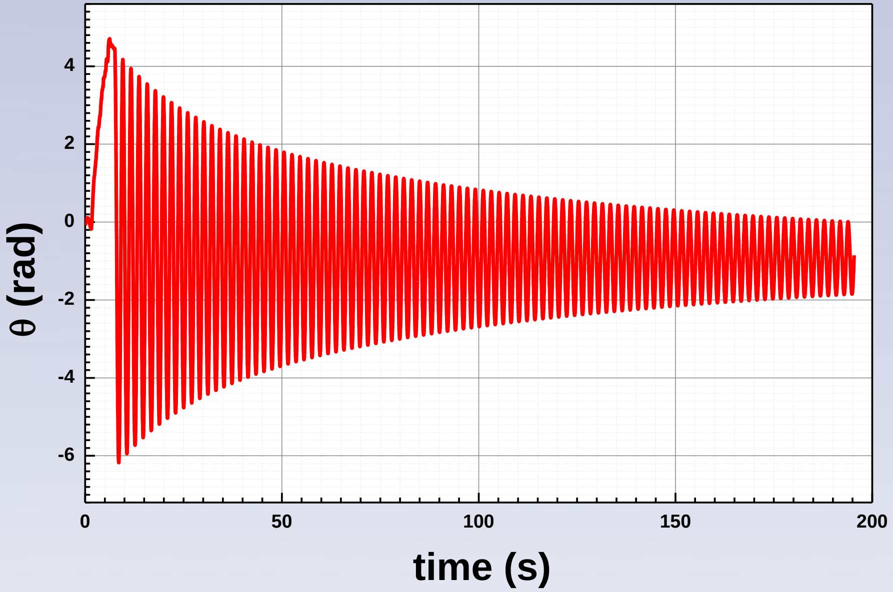
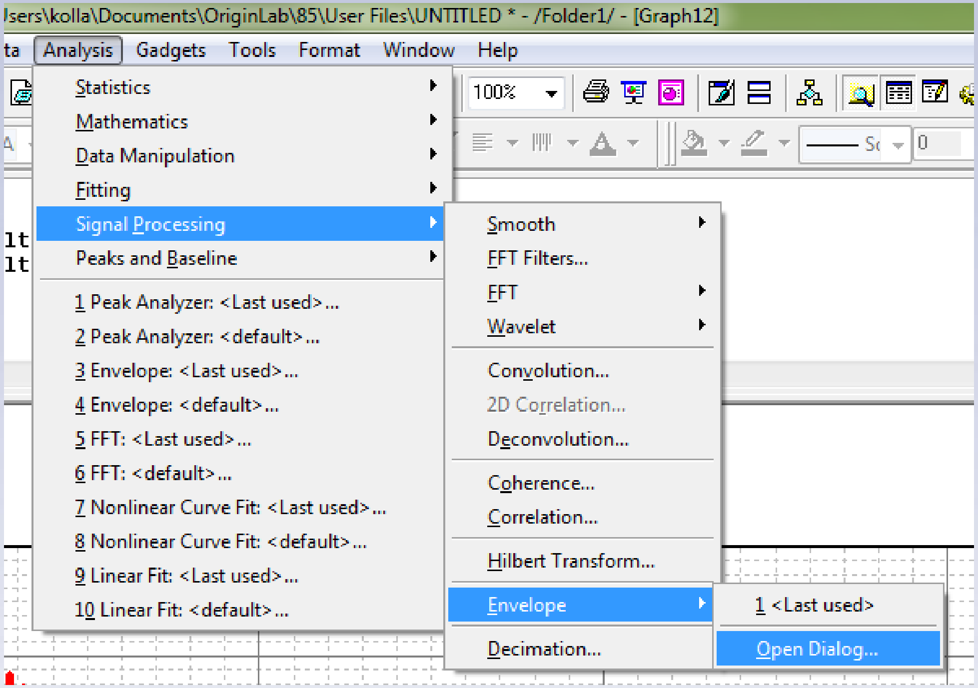
# Data analysis. Finding the peaks.




**2nd Technique: using "*Envelope*" option**

**Origin will create the worksheet with interpolated (defined for the same x's as the raw data) "envelope" data**

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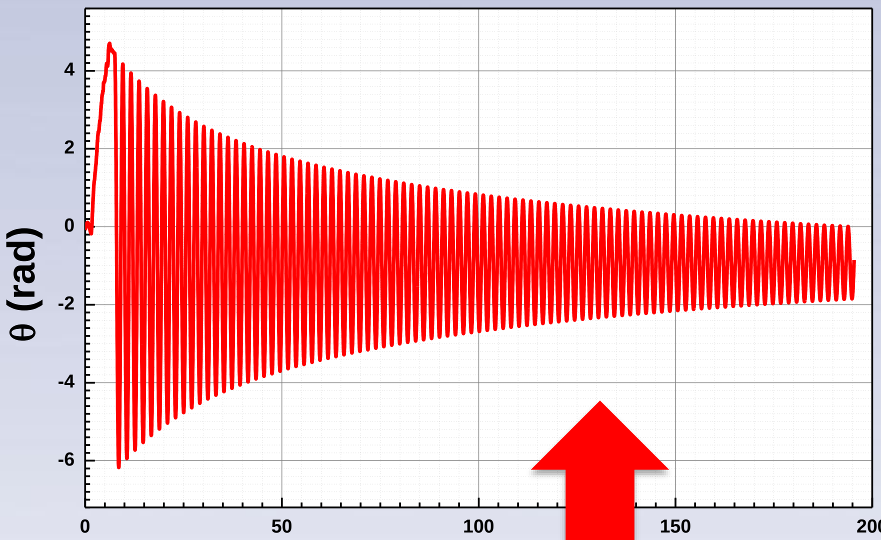
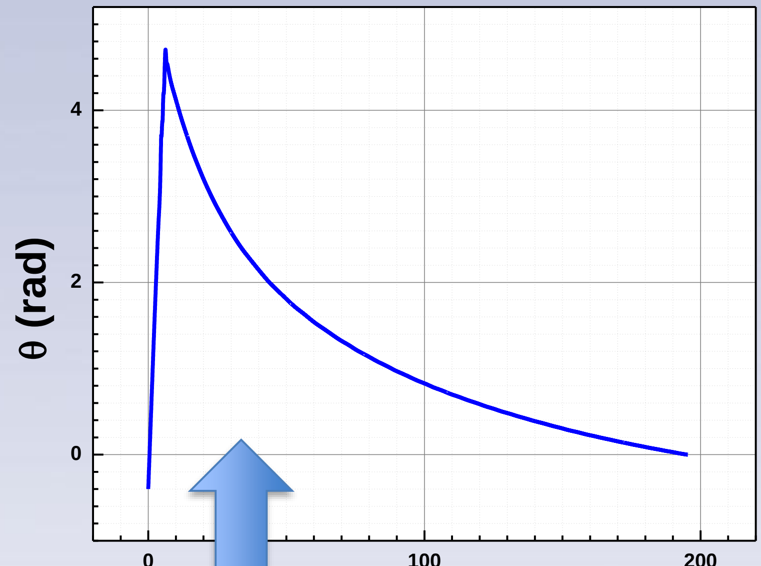
# Data analysis. Finding the peaks.




| | A(X1) | B(Y1) | C(Y1) | X1(X2) | Y1(Y2) |
|---|---|---|---|---|---|
| ts | | | | Upper Envelope of "Pend_theta (ra | Upper Envelope of "Pend_theta (radians)" |
| ie | Time | Pend_theta (radians) | Motr_theta (radians) | Envelope X 1 | Envelope Y 1 |
| 1 | 0 | -0.0813 | 0 | 0 | -0.39883 |
| 2 | 0.02 | -0.07286 | 0 | 0.02 | -0.38144 |
| 3 | 0.04 | -0.06443 | 0 | 0.04 | -0.36403 |
| 4 | 0.06 | -0.05522 | 0 | 0.06 | -0.34662 |
| 5 | 0.08 | -0.04679 | 0 | 0.08 | -0.32921 |

**Original data**

**Envelope data**

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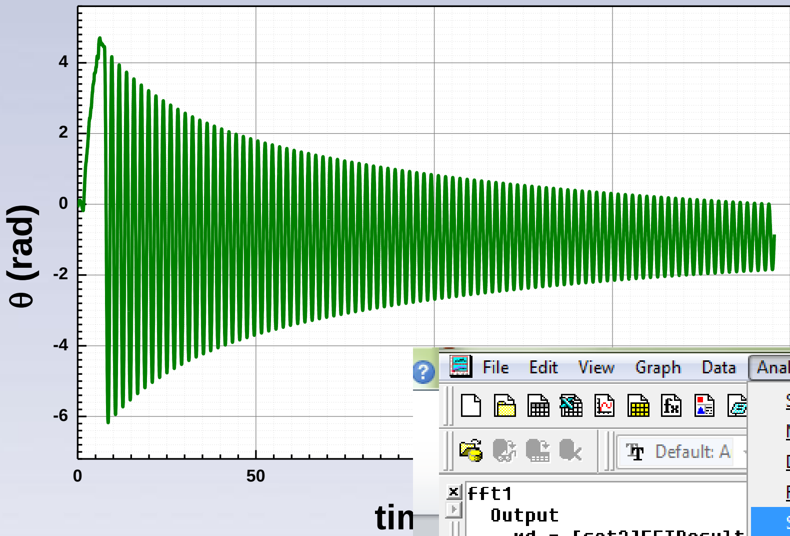
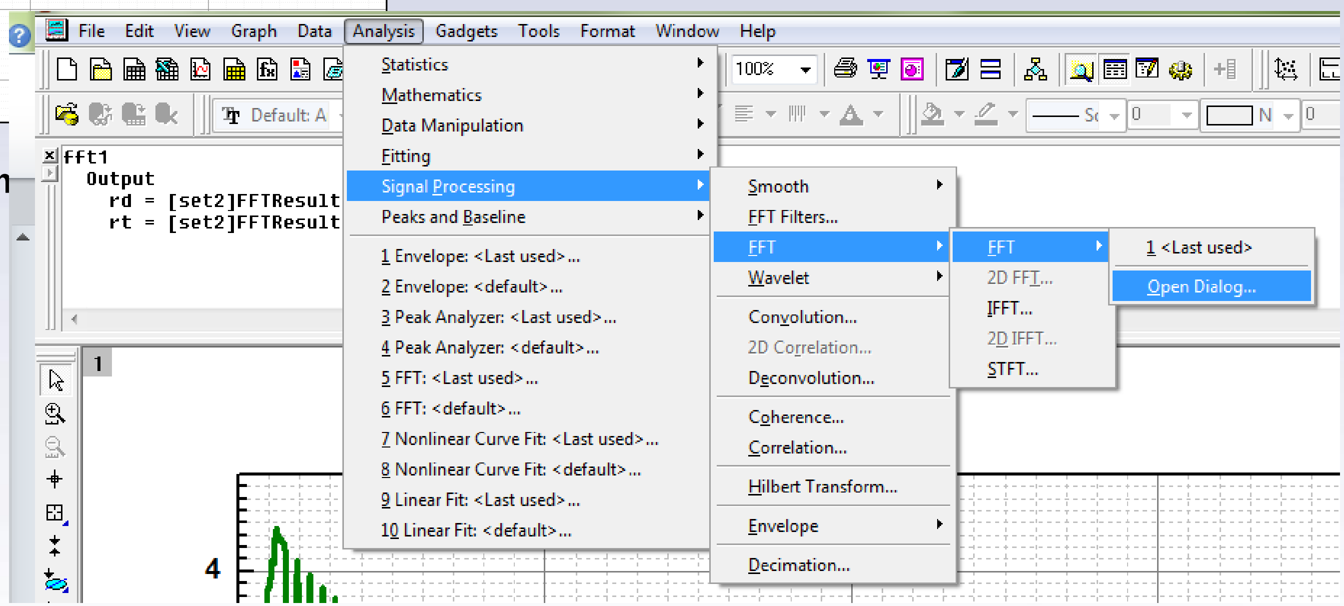
# Data analysis. FFT.

**All these quasi periodic data can be analyzed using Fast Fourier Transform**




**Our goal:** **find the resonance frequency of the pendulum**

***Origin* window**

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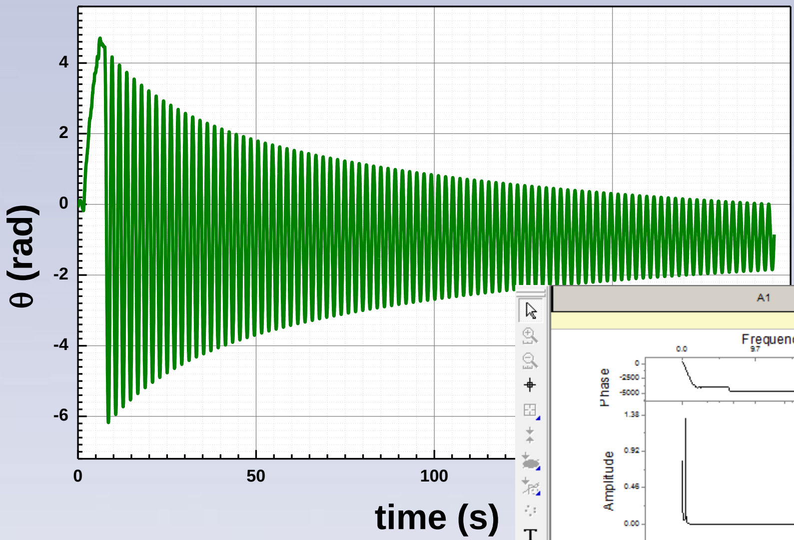
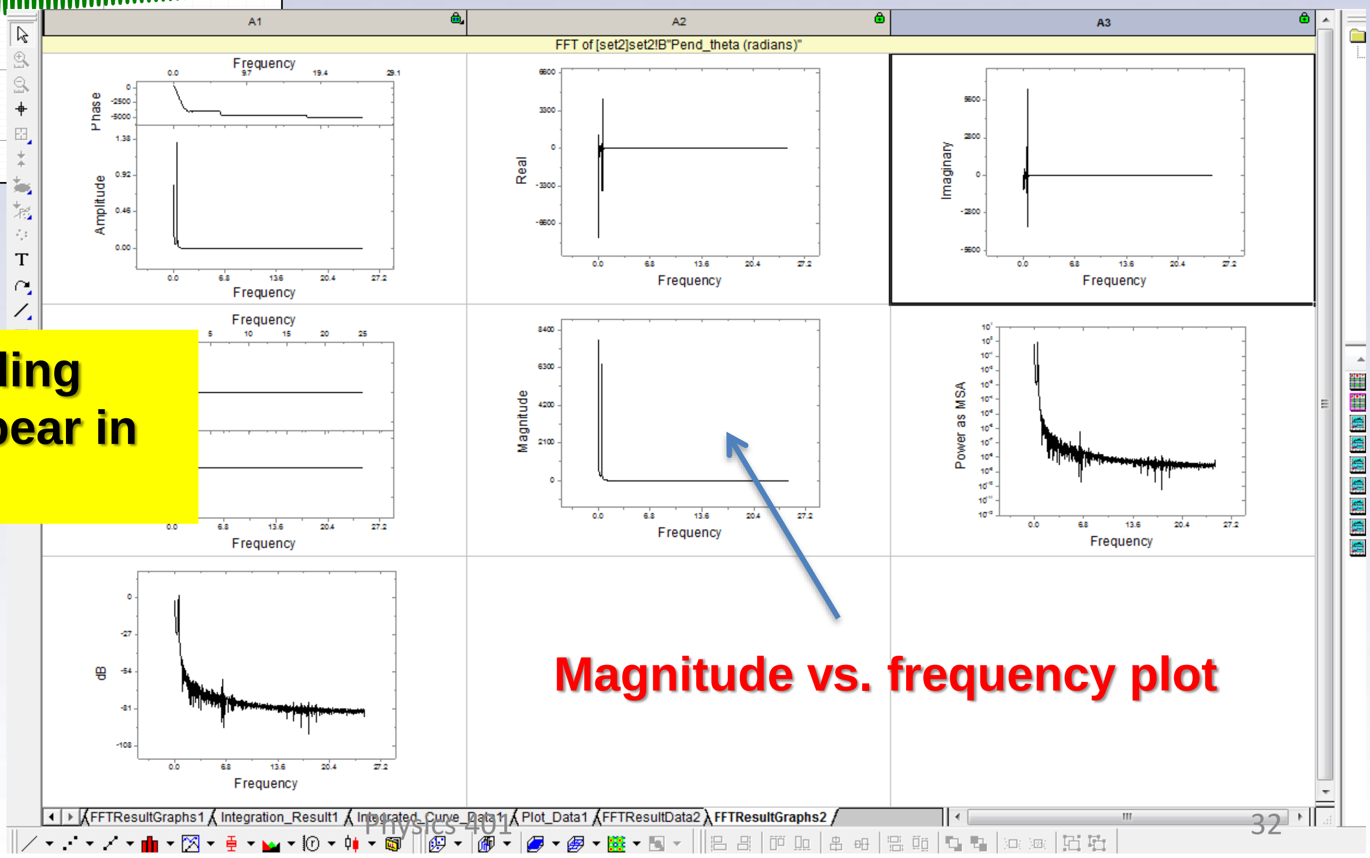
# Data analysis. FFT.




The results of FFT you can find in the same Workbook which contains the raw data




**Click on corresponding graph and it will appear in separate window**

Magnitude vs. frequency plot

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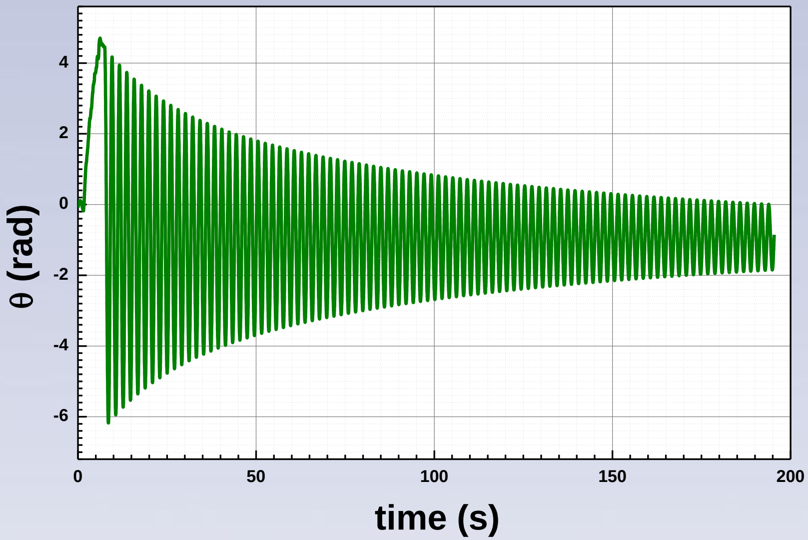
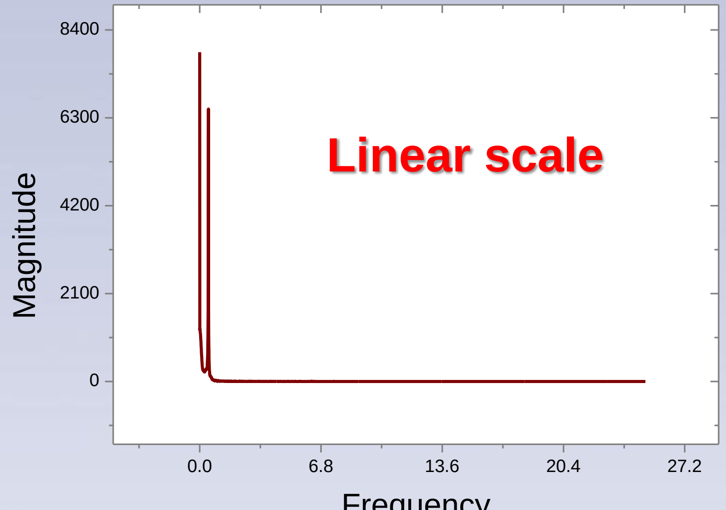
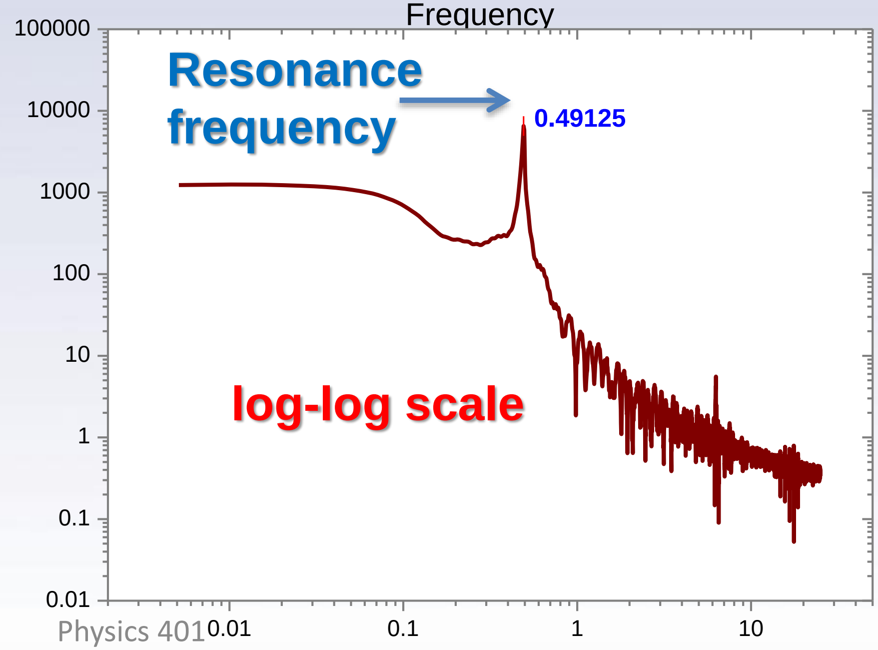
# Data analysis. FFT.




**Spectrum better to present in log-log scale**

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# Appendix. Some comments on oil drop experiment error analysis.

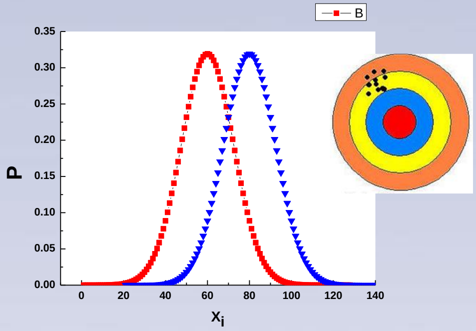
**Result of measurement** → $X_{meas}$

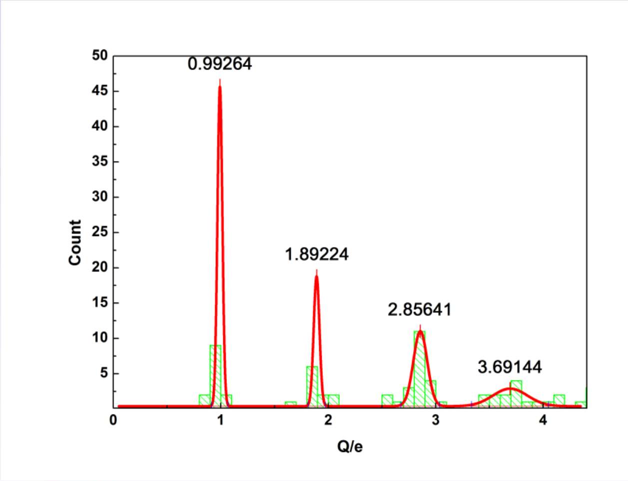
**Systematic error** → $e_s$

$$X_{meas} = X_{true} + e_s + e_r$$

**Correct value** → $X_{true}$

**Random error** → $e_r$

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# Appendix. Some comments on oil drop experiment error analysis.

**Systematic error**

$$X_{meas} = X_{true} + e_s + e_r$$

$$Q = F \bullet S \bullet T = \left(\frac{1}{f_c^{3/2}}\right)\frac{9\pi d}{V}\sqrt{\frac{2\eta^3 x^3}{g\rho}}\frac{1}{\sqrt{t_g}}\left(\frac{1}{t_g}+\frac{1}{t_{rise}}\right)$$

$$F = \frac{1}{f_c^{3/2}} \approx 1-\left(\frac{t_g}{\tau_g}\right)^{\frac{1}{2}}$$

$$S = \frac{9\pi d}{V}\sqrt{\frac{2\eta^3 x^3}{g\rho}}$$

$$T = \frac{1}{\sqrt{t_g}}\left(\frac{1}{t_g}+\frac{1}{t_{rise}}\right)$$

$$\Delta Q = \sqrt{\left(\frac{dQ}{dF}\right)^2(\Delta F)^2+\left(\frac{dQ}{dS}\right)^2(\Delta S)^2+\left(\frac{dQ}{dT}\right)^2(\Delta T)^2} \approx \sqrt{\left(\frac{dQ}{dS}\right)^2(\Delta S)^2+\left(\frac{dQ}{dT}\right)^2(\Delta T)^2}$$

$$= \sqrt{(FT)^2(\Delta S)^2+(FS)^2(\Delta T)^2} = Q\sqrt{\left(\frac{\Delta S}{S}\right)^2+\left(\frac{\Delta T}{T}\right)^2}$$

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# Appendix. Some comments on oil drop experiment error analysis.

**Systematic error**

$$X_{meas} = X_{true} + e_s + e_r$$

$$\Delta Q \approx Q\sqrt{\left(\frac{\Delta S}{S}\right)^2 + \left(\frac{\Delta T}{T}\right)^2}$$

$$\frac{\Delta S}{S} = \sqrt{\left(\frac{\Delta d}{d}\right)^2 + \left(\frac{\Delta V}{V}\right)^2 + \left(\frac{3}{2}\frac{\Delta x}{x}\right)^2 + \left(\frac{3}{2}\frac{\Delta \eta}{\eta}\right)^2 + \left(\frac{1}{2}\frac{\Delta \rho}{\rho}\right)^2 + \left(\frac{1}{2}\frac{\Delta g}{g}\right)^2} \approx \sqrt{\left(\frac{\Delta d}{d}\right)^2 + \left(\frac{3}{2}\frac{\Delta x}{x}\right)^2}$$

$$\Delta T = \sqrt{\left(\frac{3/2}{t_g^{\ 5/2}} + \frac{1/2}{t_g^{\ 3/2}}\frac{1}{t_{rise}}\right)^2 \Delta t_g^{\ 2} + \left(\frac{1}{t_g^{\ 1/2}}\frac{1}{t_{rise}^{\ 2}}\right)^2 \Delta t_{rise}^{\ 2}}$$

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**Result of measurement** → $X_{meas}$
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$$X_{meas} = X_{true} + e_s + e_r$$

**Correct value** → $X_{true}$
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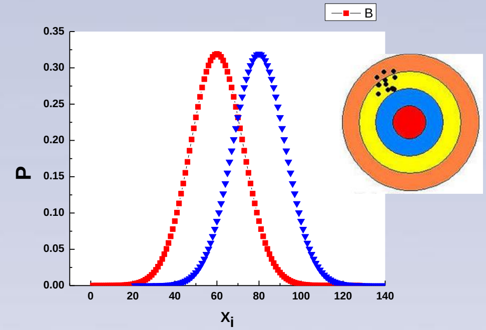


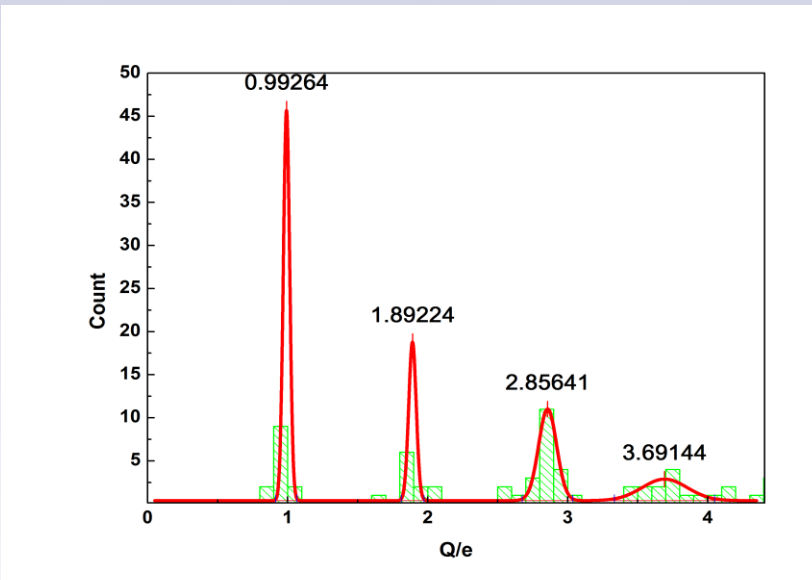


**Mean of $\{x_i\}$**

$$\mu = \frac{1}{N}\sum_{i=0}^{N-1} x_i$$

**Standard deviation of of $\{x_i\}$**

$$\sigma^2 = \frac{1}{N-1}\sum_{i=0}^{N-1}(x_i-\mu)^2$$

**Standard deviation of mean**

$$\sigma_X = \frac{\sigma}{N^{1/2}}$$

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